Test #1 MATH 200 ANSWERS

(1)

A) $W = \vec{a}\cdot\vec{b} = \langle 1,0,-1\rangle \cdot \langle 2,-1,-2\rangle = \boxed{4}$

B) $\Phi = \vec{a}\cdot(\vec{b}\times\vec{c}) = \begin{vmatrix} 1 & 0 & -1 \\ 2 & -1 & -2 \\ 1 & 1 & 1 \end{vmatrix} = 1+0-3 = \boxed{-2}$

C) $\text{proj}_{\vec{c}}\,\vec{b} = \dfrac{\langle 2,-1,-2\rangle\cdot\langle 1,1,1\rangle}{3}\langle 1,1,1\rangle = \boxed{-\tfrac{1}{3}\langle 1,1,1\rangle}$

D) $\text{Comp}_{\vec{b}}\,\vec{a} = \langle 1,0,-1\rangle \cdot \dfrac{\langle 2,-1,-2\rangle}{3} = \boxed{\dfrac{4}{3}}$

E) $\text{Area} = \left|\,|\vec{b}\times\vec{c}|\,\right| = |\langle 1,-4,3\rangle| = \boxed{\sqrt{26}}$

F) $\text{Vol} = \boxed{2}$ using the work from part B.

(2)

$$\begin{array}{r} \vec{PQ} = \langle 2,1,1\rangle \\ \times\ \vec{PR} = \langle 0,1,0\rangle \\ \hline \langle -1,0,2\rangle \end{array} \Rightarrow -x+2z = \langle -1,0,2\rangle\cdot\langle 1,1,1\rangle \Rightarrow \boxed{-x+2z=1}$$

(3)

$t^3=-1 \Rightarrow t=-1$; $t^3=27 \Rightarrow t=3$ 1 point error if just t

$$\vec{r}\,'(t) = \langle t, t^2, t\rangle \Rightarrow S = \int_{-1}^{3}\sqrt{t^2+t^4+t^2}\,dt = \int_{-1}^{3}|t|\sqrt{2+t^2}\,dt$$

$$= -\int_{-1}^{0} t\sqrt{1+t^2}\,dt + \int_0^3 t\sqrt{1+t^2} = +\int_2^3 \tfrac{1}{2}\sqrt{u}\,du + \int_2^{11}\tfrac{1}{2}\sqrt{u}\,du = \tfrac{1}{2}\left(\tfrac{2}{3}u^{3/2}\Big|_2^3 + \tfrac{2}{3}u^{3/2}\Big|_2^{11}\right)$$

$u=t^2+2$ $\quad$ $u=t^2+2$

$$= \frac{1}{3}\left(3\sqrt{3}-2\sqrt{2}+11\sqrt{11}-2\sqrt{2}\right) = \boxed{\frac{1}{3}\left(3\sqrt{3}+11\sqrt{11}-4\sqrt{2}\right)}$$

1 point error answer: $\frac{1}{3}(11\sqrt{11}-3\sqrt{3})$

(4)

$\hat{T}\cdot\hat{T} = 1 \Rightarrow \dfrac{d(\hat{T}\cdot\hat{T})}{dt} = 0 \Rightarrow \hat{T}'\cdot\hat{T} + \hat{T}\cdot\hat{T}' = 0 \Rightarrow 2\hat{T}\cdot\hat{T}' = 0 \Rightarrow \hat{T}\cdot\hat{T}' = 0$,

so $\hat{T}$ and $\hat{T}'$ are perpendicular.

(5)

A) $\boxed{\vec{r}(t) = \langle 1,1,1\rangle + t\langle 2,1,1\rangle;\ 0\le t\le 1}$

B) $\boxed{\vec{r}(t) = \langle 3,2,5\rangle + 3\langle \cos t, \sin t, 0\rangle;\ 0\le t\le 2\pi}$

C) $\boxed{\vec{r}(t) = \langle t, t-t^2\rangle \quad 0\le t\le 3}$

D) $x = z-2y$, so $\boxed{0\le t\le 2\pi}$

$\boxed{\vec{r}(t) = \langle 2\sin t - 4\cos t, 2\cos t, 2\sin t\rangle}$

E) $\boxed{\vec{r}(t) = \langle \tfrac{1}{2}\cos t, -\tfrac{1}{3}\sin t\rangle \quad 0\le t\le 2\pi}$

(6)

A)
$$\begin{array}{r} \langle 2,1,2\rangle \\ \times\ \langle 4,0,5\rangle \\ \hline \langle 5,-2,-4\rangle \end{array}$$

$\langle 5,-2,-4\rangle\cdot\langle 1,2,3\rangle = 5-4-12 = -11 \Rightarrow \boxed{5x-2y-4z=-11}$

B) $\kappa = \dfrac{\sqrt{25+4+16}}{27} = \dfrac{3\sqrt{5}}{27} = \boxed{\dfrac{\sqrt{5}}{9}} = \kappa$

$\boxed{R = \dfrac{9}{\sqrt{5}}}$

C) $\boxed{\hat{T} = \dfrac{\langle 2,1,2\rangle}{3}}$ $\quad \vec{e} = \langle 4,0,5\rangle - \dfrac{18}{9}\langle 2,1,2\rangle = \langle 0,-2,1\rangle \Rightarrow \hat{N} = \dfrac{\langle 0,-2,1\rangle}{\sqrt{5}}$

(7)

A) $\underline{u_1} = \hat{u}\cdot\hat{i} = 1\cdot 1\cdot\cos\alpha = \cos\alpha$. $\underline{u_2} = \hat{u}\cdot\hat{j} = 1\cdot 1\cdot\cos\beta = \cos\beta$. $\underline{u_3} = \hat{u}\cdot\hat{k} = 1\cdot 1\cdot\cos\gamma = \cos\gamma$.

B) $|\hat{u}| = 1 \Rightarrow u_1^2+u_2^2+u_3^2 = 1 \Rightarrow \cos^2\alpha+\cos^2\beta+\cos^2\gamma = 1$ by part A.