Math 200 Test #1 100 points **Name:**____

Always show organized work.

1) Find a position function for the following curves. Specify the domain of the parameter. (15 points)

1A) The curve is a line segment that starts at (2, 4, 0) when the parameter t = 0 and ends at $(0,1,-1)$ when t = 0.5.

1B) The curve is the piece of the ellipse $\frac{x^2}{9}+\frac{y^2}{25}=1$ that starts at (3,0) and rotates clockwise to $\left(0,5\right)$.

1C) The curve is the path on the graph of $y=\sqrt{x}$ that starts at $\left(25,5\right)$ and ends at $(1,1)$.

2) **Find and simplify** the arc length of $\vec{r}(t)=\left\langle 2t,\cosh(2t),\sinh(2t)\right\rangle$ for $0\le t\le \ln(\sqrt{2\pi})$. (10 points)

3) Find the equation of the plane, in ax + by + cz = d form, that contains the three points P = $(5,3,1)$, Q = $(2,6,2)$, and R = $(4,4,4)$. (10 points)

4) Let $\vec{a} = \langle 1,2,2 \rangle$ for all parts of this problem. (15 points)

4A) Find the angle between $\vec{a}$ and $\hat{\mathbf{i}} + \hat{\mathbf{j}}$.

4B) What is $(\hat{\mathbf{i}} + \hat{\mathbf{j}})_{\vec{a}\|}$, the projection of $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ onto $\vec{a}$?

4C) What is the error vector $(\hat{\mathbf{i}} + \hat{\mathbf{j}})_{\vec{a}\perp}$?

5) Find the work done by force $\vec{F} = \langle 1, -1, 1 \rangle$ on a particle with displacement $\vec{d} = \langle 4, -2, -5 \rangle$. (3 points)

6) Find the flux of $\vec{F} = \langle 1, -1, 1 \rangle$ through the parallelogram spanned by $\vec{a} = \langle 1, 2, 2 \rangle$ and $\vec{b} = \langle 1, 2, 4 \rangle$. Positive orientation is from $\vec{a} = \langle 1, 2, 2 \rangle$ to $\vec{b} = \langle 1, 2, 4 \rangle$. (7 points)

7) Sketch the **surface** corresponding to $x^2 - y^2 + z^2 = 0$. Label axes using the positive orientation. (5 points)

8) Convert the rectangular coordinates $\left(-1, \sqrt{3}, 2\right)$ to **cylindrical and to spherical** coordinates. Show organized work. (10 points)

9) Find a parameterization for the line that contains the point P = (1, 2, 1) and is parallel to the line parameterized by $\vec{\alpha}(t) = \langle 1+2t, 3-4t, 5+6t \rangle$. (5 points)

10) Find and simplify $\left.\dfrac{d(\vec{f}(t)\cdot\vec{g}(t))}{dt}\right|_{t=0}$ if $\vec{f}(0) = \langle 1,2,3 \rangle$, $f'(0) = \langle -2,1,2 \rangle$, and $\vec{g}(t) = \left\langle \cos(\pi e^t), \sin(\pi e^t), e^t \right\rangle$.

Show organized work. (10 points)

11) The curve C is parameterized by $\vec{r}(t)$; $r(4) = (1,3,5)$, $\vec{r}'(4) = \langle 1,-2,2 \rangle$ and $\vec{r}''(4) = \langle -2,1,2 \rangle$.

11A) Find the curvature of C at $r(4)$. (6 points)

11B) Find the equation of the osculating plane for C at $r(4)$, and write it in standard form. (4 points)