Test #1 100 points Math 200 Name:________________________________

Show work

1) Evaluate each limit or show why it does not exist. Defend your answers. (8 points)

1A) $\lim\limits_{(x,y)\to(0,0)} \dfrac{1-e^{\sqrt{x^2+y^2}}}{\sqrt{x^2+y^2}}$

1B) $\lim\limits_{(x,y)\to(3,3)} \dfrac{2x-y-3}{x-y}$

2) Find the mass of a wire with density $\delta(x,y)=\dfrac{xy}{\sqrt{1+\cos^2(x)}}$ grams per cm if the wire is located on the path $y=\sin(x)$ from the point $(0,0)$ to the point $(\pi,0)$. (5 points)

3) Find and evaluate an integral representing the work done by $\vec{F} = \langle -y, x, z + x \rangle$ on a particle that moves on the curve parameterized by $\vec{r}(t) = \langle \sin t, \sin t, \cos t \rangle$ from $t = 0$ to $t = \pi$. (5 points)

4) Evaluate I = $\int_{C_1 + C_2} 4xy\, dx + 2y\, dy$ if C_1 is a line segment from (1, 3) to (1, 1) and C_2 lies on $y = x^2$ from (1, 1) to (3, 9). (5 points)

5) $\vec{f}(t) = \langle \cos(t), \tan(t) \rangle$, $\vec{g}(\pi) = \langle 1, 2 \rangle$, and $g'(\pi) = \langle 5, 8 \rangle$. Find $\left(\vec{f} \cdot \vec{g} \right)'(\pi)$. (5 points)

6A) Find all the second partial derivatives of $f(x,y,z)=e^x y-z^2$. (5 points)

6B) What is $\nabla f\left(0,1,2\right)$ if $f(x,y,z)=e^x y-z^2$? (2 points)

7A) What is the divergence of $\vec{F}(x,y,z)=\left\langle e^x y,z^2,x^2\right\rangle$? (1 point)

7B) What is the curl of $\vec{F}(x,y,z)=\left\langle e^x y,z^2,x^2\right\rangle$? (2 points)

8) Find the equation of the plane that contains the points P = $(1,1,0)$, Q = $(1,0,1)$, and R = $(0,1,2)$. Write your final answer in standard form. (6 points)

9) Let $\vec{a} = \langle 1,2,2 \rangle$, $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$, and $\vec{c} = \langle = -2,1,-1 \rangle$. (28points @ 4 points each)

9A) Find the area of the parallelogram spanned by $\vec{b}$ and $\vec{c}$.

9B) Find the work done on an object with displacement $\vec{a}$ by force $\vec{b}$.

9C) Find the flux of $\vec{a}$ through the parallelogram spanned by $\vec{b}$ and $\vec{c}$.

9D) Find the volume of the box spanned by $\vec{a}$, $\vec{b}$, and $\vec{c}$.

9E) Find $\vec{a}_{\vec{b}||}$, the projection of $\vec{a}$ onto $\vec{b}$

9F) Find $\vec{a}_{\vec{b}\perp}$, the normal component of $\vec{a}$ to $\vec{b}$

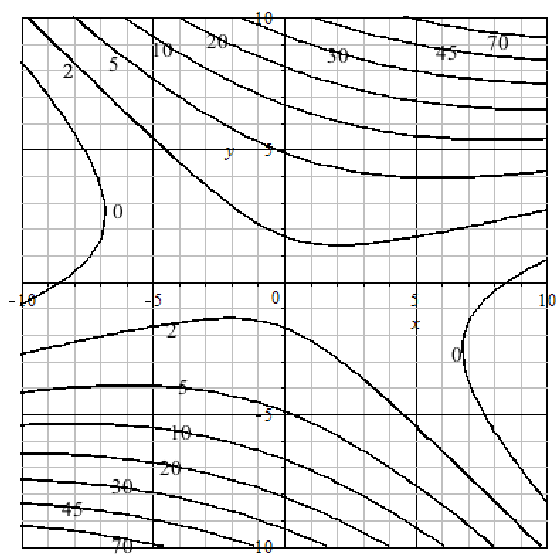
9G) Find the curvature at $r(0) = (1,0,-1)$ if $\vec{r}'(0) = \vec{b}$ and $\vec{r}''(0) = \vec{c}$.

10) Find the arc length of $\vec{r}(t) = \langle \cos t, \cosh t, \sin t \rangle$ from t = 0 to t = ln(3). (7 points)

Hint: $\cosh^2(x) - \sinh^2(x) = 1$.

11) Sketch the surface corresponding to $y = -\sqrt{z^2 + x^2}$. Label the axes with a positive orientation. (4 points)

12) Use the contour plot of f(x, y) to estimate $f_y(5,5)$.
Draw an appropriate segment. (3 points)



13) Find a parameterization in both vector form and coordinate equations form for the line that passes through the points P = (1, 2, 1) and Q = (2, 1, 2). (5 points)

14) Convert $Q = \left(2\sqrt{2}, \frac{\pi}{4}, \frac{\pi}{2}\right)_S$ to rectangular coordinates. (5 points)

15) Sketch the region for which $r \leq z$ and $\rho \leq 1$. Write a description if you can't sketch it. (4 points)