Test 1 100 points Math 200 Name: ____________________

1. (4 points) Find the equation of the plane in standard form that contains the point $(1, 2, 4)$ and is parallel to the lines $\vec{\alpha}(t) =< 4 - t, 3t + 7, 2t >$ and $x = 1, y = 2 + t, z = 7 - t$.

2. Let $\vec{u} = 2\hat{\imath} - \hat{\jmath} - 2\hat{k}$, $\vec{v} =< 1, 0, -1 >$, and $\vec{w} =< 1, 1, 1 >$. You may use work from one part in other parts.

   (a) (4 points) Find the area of the parallelogram spanned by $\vec{u}$ and $\vec{w}$

   (b) (3 points) Find the flux of $\vec{v}$ through the parallelogram spanned by $\vec{u}$ and $\vec{w}$.

   (c) (4 points) Find $\vec{u}_{\|\vec{w}}$ and $\vec{u}_{\perp\vec{w}}$.

3. Let $\vec{\alpha}(t) = \left\langle \frac{t^2}{2}, \frac{t^3}{3}, \frac{t^2}{2} \right\rangle$ for all parts of this problem. You do not need to repeat work.

(a) (8 points) Find the arc length of the trace of $\vec{\alpha}(t)$ from $t = 1$ to $t = 3$.

(b) (5 points) Find the equation of the osculating plane for the trace of $\vec{\alpha}(t)$ at $t = 1$.

(c) (5 points) Find the curvature for the trace of $\vec{\alpha}(t)$ at $t = 1$.

4. (5 points) Find a parameterization for the circle in the plane $z = 5$ with center at $(3, 2, 5)$ and radius 4.

5. (6 points) Sketch $z = x^2 - y^2$ with positively oriented xyz-axes and then sketch the corresponding contour plot with at least three labeled level curves that describe the entire surface.

6. (8 points) Find $f'(1)$ if $f(t) = \vec{p}(t) \cdot \left\langle \ln(t), \cos(3\pi t), e^t \right\rangle$, $\vec{p}(1) = \langle 1, 2, 3\rangle$, and $\left.\dfrac{d\vec{p}}{dt}\right|_{t=1} = \langle -1, 0, 2\rangle$. Show work to defend your answer.

7. Let $f(x, y) = x \ln(xy)$. You do not need to repeat work.

   (a) (4 points) Find the gradient of $f(x, y)$; that is, find ∇f.

   (b) (6 points) Find and simplify the second partial derivatives of $f(x, y)$.

8. Find the limit if possible, or prove that the limit does not exist. Show organized work to defend your answer.

   (a) (4 points) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^4 + y^2}$

   (b) (4 points) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\cos\left(\sqrt{x^2 + y^2}\right) - 1}{\sqrt{x^2 + y^2}}$

9. (8 points) Find the mass of the wire that lies on the part of $x^2 + y^2 = 16$ in the first quadrant of the xy-plane if the density of the wire is $\delta(x, y) = xy$.

10. (8 points) Use an integral to find the work done by the vector field $\vec{F}(x, y) = \dfrac{-y\hat{\imath} + x\hat{\jmath}}{\sqrt{x^2 + y^2}}$ on a particle that moves on a line from $(1, 2)$ to $(2, 4)$.

11. (5 points) Sketch the region for which $r^2 \geq z^2$ and $\rho \leq 1$. Write a description if your sketch needs help. r is the cylindrical coordinate and ρ is the spherical coordinate.

12. (5 points) Convert the point $Q = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}, \sqrt{3}\right)$ to cylindrical and spherical coordinates.

13. (4 points) Let $\vec{u} = \langle u_1, u_2, u_3 \rangle$ be a unit vector so that $\|\vec{u}\| = 1$. Also, let the angles between $\vec{u}$ and $\hat{\imath}$, $\vec{u}$ and $\hat{\jmath}$, $\vec{u}$ and $\hat{k}$ be α, β, and γ respectively. Prove $\cos^2(\alpha) + cos^2(\beta) + cos^2(\gamma) = 1$.