Test #1 Answers Math 200

① A) $W = \vec{v} \cdot \vec{w} = \langle 1,2,1\rangle \cdot \langle 2,2,1\rangle = 7$

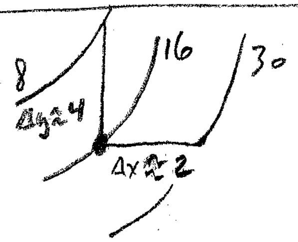
B) $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 2 & 2 & 1 \end{vmatrix} = \langle 0,1,-2\rangle \Rightarrow \boxed{y-2z=0}$

C) $\text{Area } \Delta = \frac{1}{2}\left|\langle 0,1,-2\rangle\right| = \boxed{\frac{1}{2}\sqrt{5}}$

D) $\text{flux} = \langle 1,5,2\rangle \cdot \langle 0,1,-2\rangle = 5-4 = \boxed{1}$

E) $\text{comp}_{\vec{w}}\vec{v} = \frac{\vec{v}\cdot\vec{w}}{|\vec{w}|} = \boxed{\frac{7}{3}}$

② $\vec{r}(t) = \langle 2,5,5\rangle + 2\langle \cos t, \sin t, 0\rangle$ or $\boxed{\begin{aligned} x &= 2+2\cos t \\ y &= 5+2\sin t \\ z &= 5 \end{aligned}}$

③ 

$\Rightarrow f_x(4,0) \approx \frac{30-16}{2} = \frac{14}{2} = \boxed{7}$

$f_y(4,0) \approx \frac{8-16}{4} = \frac{-8}{4} = \boxed{-2}$

④ $f(x,y) = \cos(xy) + e^{2xy} \Rightarrow \boxed{f_x = -y\sin(xy) + 2ye^{2xy}}$

$\Rightarrow \boxed{f_{xx} = -y^2\cos(xy) + 4y^2e^{2xy}}$ and $\boxed{f_{xy} = f_{yx} = -\sin(xy) - xy\cos(xy) + 2e^{2xy} + 4xye^{2xy}}$

Symmetry $\Rightarrow \boxed{f_y = -x\sin(xy) + 2xe^{2xy}}$ and

$\boxed{f_{yy} = -x^2\cos(xy) + 4x^2e^{2xy}}$

⑤ $\vec{PQ} = \langle 2,1,3\rangle - \langle 0,1,-1\rangle = \langle 2,0,4\rangle$, use $\underline{\langle 1,0,2\rangle}$.

$\vec{PR} = \langle -1,2,0\rangle - \langle 0,1,-1\rangle = \underline{\langle -1,1,1\rangle}$.

$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 2 \\ -1 & 1 & 1 \end{vmatrix} = \langle -2,-3,1\rangle$

$\Rightarrow -2x-3y+z = \langle -2,-3,1\rangle \cdot \langle 0,1,-1\rangle \Rightarrow \boxed{-2x-3y+z=-4}$ or $\boxed{2x+3y-z=4}$

⑥ line: $\vec{r}(t) = \langle 6,1,-2\rangle + t\langle 2,4,-3\rangle \Rightarrow \begin{aligned} x &= 6+2t \\ y &= 1+4t \\ z &= -2-3t \end{aligned}$, so the line intersects the plane

if $3(6+2t) - 4(1+4t) + (-2-3t) = -1$

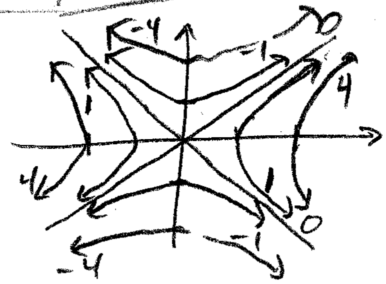
$\Rightarrow 12-13t = -1 \Rightarrow t = 1$. So the line intersects the plane at

$\boxed{\vec{r}(1) = \langle 8,5,-5\rangle}$

(7) $\langle 1,-1,1\rangle \cdot \langle 1,1,1\rangle = \sqrt{3}\sqrt{3}\cos\theta \Rightarrow \boxed{\cos\theta = \frac{1}{3}}$.

(8) $z = x^2 - y^2$

z	curve
± 1	$\pm 1 = x^2 - y^2$
± 4	$\pm 4 = x^2 - y^2$
0	$y = \pm x$



(9) $W = \int_0^{3\pi/2} \langle 3, \cos t, \sin t\rangle \cdot \langle -\sin t, \cos t, 1\rangle \, dt$

$= \int_0^{3\pi/2} -3\sin t + \cos^2 t + \sin t \, dt = (2\cos t|_0^{3\pi/2} + \int_0^{3\pi/2} \frac{1+\cos 2t}{2} \, dt$

$= -2 + \frac{3\pi}{4} + (\frac{1}{2} \frac{\sin 2t}{2}|_0^{3\pi/2} = \boxed{\frac{3\pi}{4} - 2}$

(10) Mass $= \int_1^2 8t\sqrt{1+4t^2}\, dt$ if we use C: $x = t$, $y = t^2$, $1 \le t \le 2$

$u = 1+4t^2$, $du = 8t$ $\Rightarrow M = \int_5^{17} \sqrt{u}\, du = \boxed{\frac{2}{3}(17^{3/2} - 5^{3/2})}$ $\quad \vec{r}'(t) = \langle 1, 2t\rangle \Rightarrow \frac{ds}{dt} = \sqrt{1+4t^2}$.

(11) $\int_C = \int_{C_1} + \int_{C_2} + \int_{C_3}$

C_1: $dy = 0$, $y = 0$

C_2: $y = -2x+2$, $1 > x > 0$

C_3: $x = 0$, $dx = 0$

$\int_{C_1} = \int_0^1 2x\, dx = \underline{\underline{1}}$

$\int_{C_2} = \int_1^0 (2x + 6x - 6)\, dx + -2x\, dx = 6(\frac{x^2}{2} - x|_1^0 = \underline{\underline{3}}$

$\int_{C_3} = 0$

$\Rightarrow \int_C = \boxed{4}$

(12) a) $\vec{r}'(t) = \langle -2\sin(2t), 2\cos(2t), 1\rangle \Rightarrow \vec{r}''(t) = \langle -4\cos(2t), -4\sin(2t), 0\rangle$

$\Rightarrow \vec{v}(\frac{\pi}{2}) = \langle 0, -2, 1\rangle \Rightarrow v = \sqrt{5}$, and $\vec{a}(\frac{\pi}{2}) = \langle 4, 0, 0\rangle$.

Then $\vec{v} \times \vec{a} = \langle 0, 4, 8\rangle \Rightarrow \kappa = \frac{4\sqrt{5}}{5\sqrt{5}} = \boxed{\frac{4}{5}}$.

b) $\vec{v} \cdot \vec{a} = 8\sin(2t)\cos(2t) - 8\sin(2t)\cos(2t) = 0 \Rightarrow \boxed{\vec{v} \perp \vec{a}}$ for all t

(13) $\frac{d\, \vec{r} \times \vec{v}}{dt} = \vec{0} \Rightarrow \vec{r}' \times \vec{v} + \vec{r} \times \vec{v}' = \vec{0} \Rightarrow \vec{v} \times \vec{v} + \vec{r} \times \vec{a} = \vec{0}$. But $\vec{v} \times \vec{v} = 0$ since they are parallel, so $\vec{r} \times \vec{a} = 0 \Rightarrow \vec{r}$ and $\vec{a}$ are parallel ∎