Test #1 100 points Math 200 Name:_______________________________________

**Simplify your final answers. Show organized work. Defend all answers.**

1) Find an integral that represents the work done by the force $\vec{F} = \langle x, -z, y \rangle$ on a particle that moves on the path parameterized by $\vec{r}(t) = \langle 2t, 3t, -t^2 \rangle$ from $-1 \le t \le 1$, and then **evaluate the integral**. (10 points)

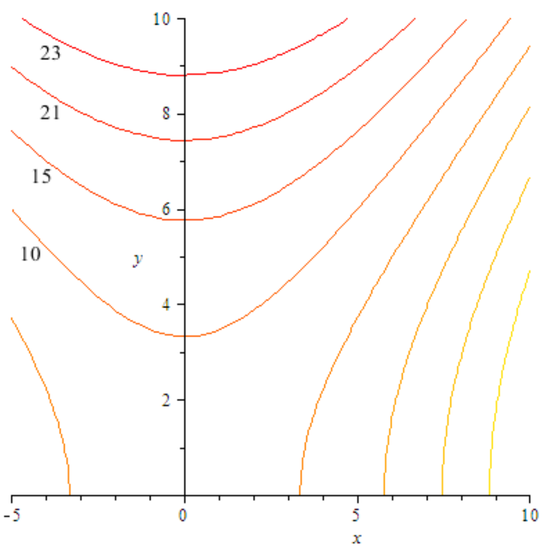
2) Find an integral that represents the mass of the wire that lies on the line segment from $(1,4,3)$ to $(4,8,3)$ if its linear density is $\delta(x,y) = \frac{xyz}{15}$, and then **evaluate the integral.** (10 points)

3) Evaluate $\int_C -y\,dx + x\,dy$ if C consists of the top half of the circle $x^2 + y^2 = 4$ from $(2,0)$ to $(-2,0)$ and the line segment from $(-2,0)$ to $(-2,3)$. (10 points)

4a) Find all the second partial derivatives of $f(x, y) = xy^5 - e^{xy}$. (5 points)

4b) What is $\nabla f(2,1)$? (3 points)

5) Use the contour plot of f(x, y) to estimate $f_y(2,6)$. Draw an appropriate segment. (5 points)




6) Find the equation of the plane that contains the points P = (1, 1, 1), Q = (3, 2, 2), and R = (1, 2, 3). Write your final answer in standard form. (10 points)

7) Let $\vec{a} = \langle 2, 2, -1 \rangle$, $\vec{b} = 2\hat{i} - 4\hat{j} - 4\hat{k}$, and $\vec{c} = \langle 3, 0, 4 \rangle$ for each part in this problem. (20 points)

a) Find the component of $\vec{b}$ on $\vec{c}$.

b) Find the area of the parallelogram determined by $\vec{a}$ and $\vec{b}$.

c) Find the volume of the box determined by $\vec{a}$, $\vec{b}$, and $\vec{c}$.

8) Assume $\hat{N}(t)$ is the unit normal vector for a smooth path $\vec{r}(t)$. Use the product rule to prove that $\hat{N}(t)$ and $\dfrac{d\hat{N}(t)}{dt}$ are perpendicular. (5 points)

9) Find the following at P = (2, 3, 1) for the curve C if $\vec{r}(t)$ is a parameterization of C with $\vec{r}(0) = \langle 2,3,1 \rangle$, $\vec{r}'(0) = \langle 1,2,2 \rangle$, and $\vec{r}''(0) = \langle 1,1,1 \rangle$. (12 points)

a) Find $\kappa$, the curvature at P, and R, the radius of the osculating circle at P.

b) Find $\hat{T}$ and $\hat{N}$.

10) Graph the two surfaces $z = x^2 + y^2$ and $z = 18 - x^2 - y^2$. Next draw the curve C where the surfaces intersect, and then find a parameterization for a particle that rotates once clockwise around the z – axis on C. (10 points)