Test #2A 100 points Math 200 Name:_____

Simplify your final answers. Show organized work. Defend all answers.

1) Given $f(x,y)$, let $g(u,w) = f(u^3 - w^3, w^3 - u^3)$. Use the chain rule to prove $w^2 \frac{\partial g}{\partial u} + u^2 \frac{\partial g}{\partial w} = 0$. Hint: find expressions for $\frac{\partial g}{\partial u}$ and $\frac{\partial g}{\partial w}$. (10 points)

2) Sketch the region of integration for I = $\int_0^1 \int_x^{2x} f(x,y)\,dydx$ and then write I as an integral expression with the order of integration switched. (10 points)

3) Find the center of mass for the rectangular lamina with vertices $[-2,2]\times[0,1]$ if its density equals y grams per square meter. (10 points)

4) Find I_0, the moment of inertia about the origin, for the lamina that is half an annulus $1 \le x^2 + y^2 \le 9$ and $y \ge 0$ if its density equals $\frac{y}{x^2 + y^2}$ grams per square meter. (10 points)

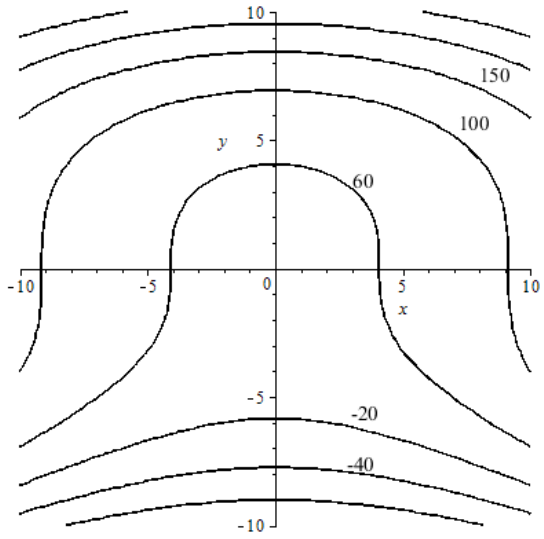
5a) Find m and w so that $\vec{F} = (mx^2 y - y^3 + 1)\hat{\mathbf{i}} + (2x^3 + wxy^2 + 2)\hat{\mathbf{j}}$ is conservative. (5 points)

5b) Find a potential function for $\vec{F}$ using a method from class and the m and w found in part a. (5 points)

5c) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ if C is the curve parameterized by $\vec{r}(t) = e^t \langle \cos t, \sin t \rangle$ for $0 \le t \le \pi$. (5 points)

6) The contour map on the right is for z = f(x, y).

6a) Sketch the $\nabla f(0,-7)$ on the contour plot. (5 points)



6b) Estimate $D_{\mathbf{i}} f(5,0)$. Show work. (5 points)

7a) What is the tangent plane to the surface $z = f(x,y)$ at the point P = (5, 3, 6) if $f(5,3) = 6$ and $\nabla f(5,3) = \langle -2,3 \rangle$? (5 points)

7b) Approximate $f(5.2, 2.9)$ using differentials if $f(5,3) = 6$ and $\nabla f(5,3) = \langle -2,3 \rangle$. (5 points)

8) Find the equation of the tangent plane to the surface $ye^z = 5 - xz$ at the point Q = $(3,5,0)$. (5 points)

9a) Is $\vec{F} = (4x^2 + y^2)\hat{\mathbf{i}} + 4y\hat{\mathbf{j}} + 2z\hat{\mathbf{k}}$ conservative? YES NO (circle one) (2 points)

9b) Is the domain of $\vec{F} = (4x^2 + y^2)\hat{\mathbf{i}} + 4y\hat{\mathbf{j}} + 2z\hat{\mathbf{k}}$ simply connected? YES NO (circle one) (2 points)

9c) Find the work done by $\vec{F} = (4x^2 + y^2)\hat{\mathbf{i}} + 4y\hat{\mathbf{j}} + 2z\hat{\mathbf{k}}$ on a particle that moves once **clockwise** around the ellipse $4x^2 + y^2 = 4$ in the plane z = 1. (6 points)

10) Find the volume of the solid that is inside the cylinder $x^2 + y^2 = 4$ between the planes $z = 0$ and $x + y + z = 3$. (10 points)