Test #2B 100 points Math 200 Name:________________________________

Simplify your final answers. Show organized work. Defend all answers.

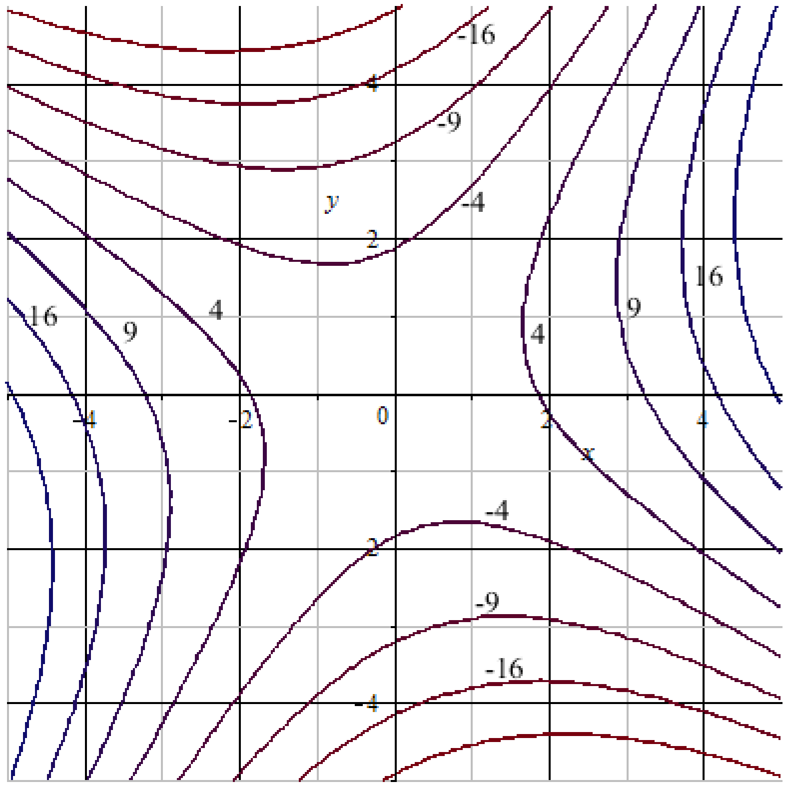
1) Find the volume of the solid inside both $x^2 + y^2 = 15$ and $x^2 + y^2 + z^2 = 16$. Simplify. (10 points)

2) Find the work done by $\vec{F} = z\hat{\mathbf{i}} + x\hat{\mathbf{j}} + y^2\hat{\mathbf{k}}$ on a particle that moves once **clockwise** around the ellipse $4x^2 + y^2 = 4$ in the plane z = 4. (10 points)

3) Evaluate I = $\int_0^1 \int_0^{\sqrt{y}} x\sqrt{1-y^2}\, dx\, dy$. Simplify your answer. (5 points)

4) The contour map on the right is for z = f(x, y).

4A) Estimate the maximum value of $D_{\hat{\mathbf{u}}} f(0,4)$. Show work. (5 points)



4B) Sketch $\nabla f(0,4)$ on the contour plot. (5 points)

5) What is the tangent plane to the surface $z = f(x, y)$ at the point P = (1, 2, 5) if $f(1,2) = 3$ and $\nabla f(1,2) = \langle -1, 2 \rangle$? Write your final answer in ax + by + cz = d form. (5 points)

6) Let $W(s,t) = F(u(s,t), v(s,t))$, where F, u, and v are differentiable. Find $W_s(1,0)$ if (10 points)

$u(1,0) = 2 \qquad v(1,0) = 1$

$u_s(1,0) = 4 \qquad v_s(1,0) = 3$

$u_t(1,0) = 6 \qquad v_t(1,0) = 5$

$F(2,1) = 8 \qquad F(2,1) = 7$

$F_u(2,1) = 10 \qquad F_v(2,1) = 9$

7A) Find m so that $\vec{F} = (-3x^2y)\hat{\mathbf{i}} + (\mathrm{m}\,x^3 + 3y^2)\hat{\mathbf{j}} + e^z\hat{\mathbf{k}}$ is a gradient field. Defend your answer. (8 points)

7B) Find a potential function for $\vec{F} = (-3x^2y)\hat{\mathbf{i}} + (\mathrm{m}\,x^3 + 3y^2)\hat{\mathbf{j}} + e^z\hat{\mathbf{k}}$ using a method from class and the m found in part a. (10 points)

7C) Evaluate $\mathrm{I} = \int_C \left[(-3x^2y)\hat{\mathbf{i}} + (\mathrm{m}\,x^3 + 3y^2)\hat{\mathbf{j}} + e^z\hat{\mathbf{k}}\right] \cdot d\vec{r}$ using m from part a and the FTCLI if C is the curve parameterized by $\vec{r}(t) = \cos(t)\langle t, \cos(t), \cos(t)\rangle$ for $0 \le t \le \pi$. (7 points)

8) Sketch the region of integration for I = $\int_0^4 \int_{-\sqrt{x}}^{x} f(x,y)\, dy\, dx$ **and** then write I as an integral expression with the order of integration switched. (10 points)

9) Find the equation of the tangent plane to the surface $2x+3y-6z=\pi\cos(x)+\sin(y)+\sin(z)$ at the point Q = (π,π,π). (10 points)

10) Approximate $f(1.3,1.9)$ using differentials if $f(1,2)=6$ and $\nabla f(1,2)=\langle 3,-4\rangle$. (5 points)