Test #2 100 points Math 200 Name:______________________________________

Circle or box, and simplify your final answers. Show work or some other defense of your answers.

1) Find an integral that represents the work done by the force $\vec{F} = \langle z, x, y \rangle$ on a particle that moves on the helix parameterized by $\vec{r}(t) = \langle t, \sin t, \cos t \rangle$ from $t = \frac{-\pi}{2}$ to $t = \frac{\pi}{2}$, and then **evaluate the integral**. (10 points)

2) Find an integral that represents the mass of the wire that lies on $x^2 + y^2 = 16$ from $(0,4)$ **clockwise** to $(4,0)$ in the xy - plane if its linear density is $\mu(x, y) = xy$, and then **evaluate the integral.** (10 points)

3) Evaluate $\int_C \left(x^2 y\right) dx + 2e^x y dy$ if C is the piecewise linear curve that moves on a line from (0, 0) to (0, 3) and then on a line from (0, 3) to (1, 3). (10 points)

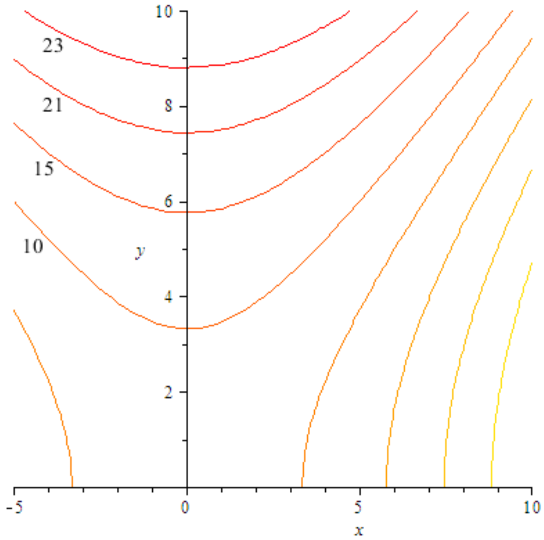
4) For what value of a is $\vec{G} = \langle 2axy + \sin(x), 3x^2 + 2 \rangle$ a conservative vector field? (5 points)

5a) Use one of the two methods discussed in class to find a potential function for the conservative vector field $\vec{F} = \dfrac{y^2}{1+x^2}\hat{\mathbf{i}} + (2y\arctan(x) - 2)\hat{\mathbf{j}}$. The domain of $\vec{F}$ is $\left\{(x,y) \middle| \dfrac{-\pi}{2} < x < \dfrac{\pi}{2}\right\}$. (10 points)

5b) Evaluate $\int_C \vec{F} \cdot \overrightarrow{dr}$ if C is a curve from $(1,0)$ to $(-1,4)$ and $\vec{F} = \dfrac{y^2}{1+x^2}\hat{\mathbf{i}} + (2y\arctan(x) - 2)\hat{\mathbf{j}}$ as in part 5a. (5 points)

6) Find $f_x(2,1)$ if $f(x,y) = g(u,v,w) = u^2v - e^{4w}$ where $u = x^2$, $v = y^2$, $w = w(x,y)$, $w(2,1) = 0$ and $w_x(2,1) = 2$. (10 points)

7) The level curves of $f(x, y)$ are shown below. Sketch the gradient vector $\nabla f(0, 5.8)$. You may assume that the point $(0, 5.8)$ lies on the level curve z = 15. (5 points)



8) Find an equation for the tangent plane to the surface $xyz = \ln(x + y + z)$ at the point $B = (0,1,0)$. (10 points)

9) Find $\dfrac{\partial^2 f}{\partial y^2}$ if $f(x, y) = \dfrac{-y}{\sqrt{x^2 + y^2}}$. (5 points)

10a) Find the directional derivative of $g(x,y,z)=\dfrac{x+y}{z}$ at the point $A=(1,1,-1)$ in the direction pointing **away** from the origin. (5 points)

10b) What is the magnitude and direction of the minimum (that is, most negative) directional derivative of $g(x,y,z)=\dfrac{x+y}{z}$ at $A=(1,1,-1)$? You may use work from 10a. (5 points)

11) Suppose $h(1,5)=3$ and $\nabla h(1,5)=\langle 2,-5\rangle$. (10 points)

11A) Use differentials to estimate $h(0.8,5.3)$.

11B) What is the equation of the tangent plane to z = h(x, y) at (1, 5, 3)? Write the answer in standard form.