Math 200 Test # 2 Answers

① $W = \int_{-\pi/2}^{\pi/2} \langle \cos t, t, \sin t\rangle \cdot \langle 1, \cos t, -\sin t\rangle \, dt = 2 + 0 - \frac{\pi}{2} = \boxed{2 - \frac{\pi}{2}}$

② $M = \int_{-\pi/2}^{0} (-16 \cos t \sin t) \cdot 4 \cdot dt = 64 \left. \frac{\cos^2(t)}{2} \right|_{-\pi/2}^{0} = \boxed{32}$

$C: \vec{r}(t) = 4\langle \cos(t), -\sin(t)\rangle, \ -\frac{\pi}{2} \le t \le 0$

③ $C_1: \begin{matrix} x=0 \\ y=t \end{matrix} \Rightarrow \int_{C_1} \vec{F} \cdot d\vec{r} = \int_0^3 2y \, dy = \underline{\underline{9}}$

$C_2: \begin{matrix} x=t \\ y=3 \end{matrix} \Rightarrow \int_{C_2} \vec{F} \cdot d\vec{r} = \int_0^1 3x^2 \, dx = \underline{\underline{1}}$

$\Rightarrow \int_C \vec{F} \cdot d\vec{r} = \boxed{10}$

④ $\frac{\partial(3x^2+2)}{\partial x} = \frac{\partial(2axy + \sin(x))}{\partial y} \Rightarrow 6x = 2ax \Rightarrow \boxed{a = 3}$

(5a) $\Phi(a,b) = \underset{\substack{x=t \\ y=0}}{\int_0^a 0 \, dx} + \underset{\substack{x=a \\ y=t}}{\int_0^b 2y \arctan(a) - 2 \, dy} = \left. y^2 \arctan(a) - 2y \right|_0^b$

$\Rightarrow \boxed{\Phi(x,y) = y^2 \arctan(x) - 2y}$

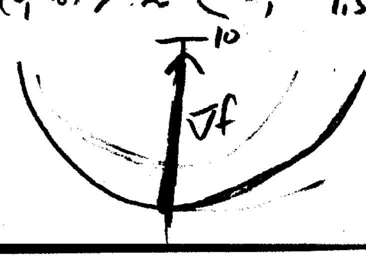
(5b) $\int_C \vec{F} \cdot d\vec{r} = \Phi(-1,4) - \Phi(1,0) = 16 \arctan(-1) - 2(4) - 0 - 0$

$= 16\left(-\frac{\pi}{4}\right) - 8 = \boxed{-4\pi - 8}$

⑥ $f_x = f_u u_x + f_w w_x = (2uv)(2x) + (-4e^{4w}) w_x$. $\begin{matrix} x=2 \\ y=1 \end{matrix} \Rightarrow \begin{matrix} u=4 \\ v=1 \\ w=0 \end{matrix}$,

so $f_x(2,1) = 8(4) - 4(2) = \boxed{24}$

⑦ $\nabla f(0,5.8) \approx \langle 0, f_y(0,5.8)\rangle \approx \langle 0, \frac{21-15}{1.5}\rangle = \underline{\underline{\langle 0,4\rangle}}$, so



(8) $g = \ln(x+y+z) - xyz \Rightarrow \nabla g = \left\langle \frac{1}{x+y+z}\langle 1,1,1\rangle - \langle yz, xz, xy\rangle \right.$

$\Rightarrow \nabla g(0,1,0) = \langle 1,1,1\rangle \Rightarrow$ the T.P. is $\boxed{x+y+z=1}$

(9) $\dfrac{\partial\left(\frac{-y}{\sqrt{x^2+y^2}}\right)}{\partial y} = \dfrac{-\sqrt{x^2+y^2} + \frac{y^2}{\sqrt{x^2+y^2}}}{x^2+y^2} = \dfrac{-x^2}{(x^2+y^2)^{3/2}}.$

Then $\dfrac{\partial\left(\frac{-x^2}{(x^2+y^2)^{3/2}}\right)}{\partial y} = \frac{3}{2}x^2(x^2+y^2)^{-5/2}\cdot 2y = \boxed{3x^2y\,(x^2+y^2)^{-5/2}}$

(10a) $\nabla g = \left\langle \frac{1}{z}, \frac{1}{z}, \frac{-x-y}{z^2}\right\rangle \Rightarrow \nabla g(1,1,-1) = \langle -1,-1,-2\rangle.$

$\hat{u} = \frac{\langle 1,1,-1\rangle}{\sqrt{3}} \Rightarrow \boxed{D_{\hat{u}}\, g(A) = 0.}$

(10b) $\min |D_{\hat{u}}\, g| = -|\nabla g| = \boxed{-\sqrt{6}}$ $\boxed{\hat{u} = \frac{\langle 1,1,2\rangle}{\sqrt{6}} \text{ is the direction.}}$

(11a) $h(0.8, 5.3) \approx 3 + 2(-.2) + (-5)(.3) = 3 - .4 - 1.5 = \boxed{1.1}$

(11b) $z - 3 = 2(x-1) - 5(y-5) \Rightarrow \boxed{2x - 5y - z = -26}$