Test #2 100 points Math 200 Name:________________________________

Show work and always simplify your answers.

1) Find an integral that represents the work done by the force $\vec{F} = \langle 3, x, y \rangle$ on a particle that moves on the helix parameterized by $\vec{r}(t) = \langle \cos t, \sin t, t \rangle$ from $t = 0$ to $t = 2\pi$, and then **evaluate the integral**. (10 points)

2) Find an integral that represents the mass of the wire that lies on $x^2 + y^2 = 16$ from $(0,4)$ **clockwise** to $(4,0)$ in the xy - plane if its linear density is $\delta(x, y) = xy$, and then **evaluate the integral.** (10 points)

3) Find an equation for the tangent plane in standard form to the surface $x + y - z - 1 = \cos(x) + \cos(y) + \cos(z)$ at $P = \left(\frac{\pi}{2}, \frac{\pi}{2}, \pi \right)$. (5 points)

4) $f(2,4)=4$ **and** $\nabla f(2,4)=\langle 5,-1\rangle$ **for both #4A and #4B.**

4A) Use differentials to estimate $f(1.8,4.3)$. (5 points)

4B) Find the equation of the tangent plane for the graph of z = f(x,y) at (2, 4, 4). (5 points)

5A) Find a value for k that makes $\vec{F}=\left\langle 4e^{x}y+kx,ke^{x}+2y\right\rangle$ a gradient field. (5 points)

5B) Use the FTCLI to evaluate $\int_C \left(4e^{x}y+kx\right)dx+\left(ke^{x}+2y\right)dy$ if k is the number found in #5A, and if C is the curve parameterized by $\vec{r}(t)=\langle 1+\cos t,1+\sin t\rangle$ for $0\le t\le \pi$. (10 points)

6) Find all of the second partial derivatives for $f(x, y) = e^{xy} - \sin(y^2)$. (10 points)

7) The temperature at any point in space is given by $T(x, y, z) = 100 - \frac{6x}{y} - \frac{6y}{z}$. If a hot bug is located at the point P = (30, 6, 2), find the direction it should move to most rapidly decrease its temperature. (10 points)

8) If $h(x, y) = g(u(x, y), v(x, y))$, $u = xy$, $v = 4x^2 - y$, and $g(u, v) = u^2 - v$, **use the chain rule** to find $h_x(1, 2)$.
Hint: x = 1, and y = 2. (5 points)

9) Sketch the region of integration for I = $\int_0^2 \int_x^{2x} f(x,y)\,dydx$ and then write I as an integral expression with the order of integration switched. (8 points)

10) Evaluate I = $\int_0^{\frac{\pi}{2}} \int_0^{\sin z} \int_0^{y\cos z} y^2 \, dxdydz$. (8 points)

11) Calculate $\iint_R x^2 \, dA$ if R is the region bounded by y = x, y = 0, and x = 2. Sketch the region R. (9 points)