Test 2 100 points Math 200 Name: ______________________________

1. For all parts of this problem let $\vec{F}(x, y, z) = (6x^2y)\hat{\imath} + (mx^3 + 2y)\hat{\jmath} + (\sin(z))\hat{k}$.

   (a) (4 points) Find m so that $\vec{F}(x, y, z)$ is a gradient field.

   (b) (10 points) Use the FTCLI to evaluate $I = \int_C \vec{F}(x, y, z) \cdot d\vec{s}$ if C is parameterized by $\vec{\alpha}(t) = \langle t^2, t\sin(t), t\cos(t) \rangle$ for $0 \leq t \leq \pi$.

2. (6 points) Evaluate $I = \int_0^{\pi/2} \int_0^y \int_0^x \cos(y + z)\, dz\, dx\, dy$.

3. For all parts of this problem, $f(x, y)$ is a differentiable function with $f(1, 3) = 4$, $\nabla f(1, 3) = -2\hat{\imath} + 5\hat{\jmath}$, and $P = (1, 3, 4)$.

   (a) (4 points) Find the equation of the tangent plane in standard form to the graph of $z = f(x, y)$ at the point P.

   (b) (4 points) Use the linear approximation for $f(x, y)$ at the point P to estimate $f(1.2, 2.7)$.

   (c) (3 points) Find the minimum value of $D_{\vec{u}} f(1, 3)$. Show some work.

4. (6 points) Let $F(x, y)$ be a differentiable function so that

$$F(4, 3) = 1 \quad F_x(4, 3) = 2 \quad F_y(4, 3) = 3$$

Let $h(x, y) = F(x^2 y, x + y)$. Find $\dfrac{\partial h}{\partial y}(2, 1)$. Show organized work.

5. (6 points) $I(M,H) = \dfrac{M}{H^2}$ is the body mass index for a person with mass M and height H. If $M = 80 \pm 2$ kg and $H = 2 \pm 0.1$ m, use differentials to estimate the maximum percentage error.

6. (6 points) Find the tangent plane in standard form for the graph of $2x + 3y - 5z = 2\sin(x) + \sin(y) + \sin(z)$ at the point (π, π, π).

7. (6 points) Sketch the region of integration for $I = \displaystyle\int_{-4}^{0} \int_{y^2/4}^{-y} f(x,y)\,dx\,dy$ and then rewrite I with the order of integration switched to $dy\,dx$.

8. (8 points) Find the net volume of the solid that is below the graph of $z = 2x\sin(xy)\cos(xy)$ if $0 \le x \le 1$ and $0 \le y \le \pi$.

9. (8 points) Find the mass of the part of the solid cylinder $x^2 + y^2 \le 4$ for which $x \ge 0$, $y \ge 0$, and $0 \le z \le 3$ if the density is $\delta(x, y, z) = z + x^2y \quad kg/m^3$.

10. (8 points) Use Green's Theorem exactly once to evaluate $I = \int_C x^2y^3\,dx + \sin(y)\,dy$ if C is the closed curve that rotates once counter-clockwise about the triangle with vertices at (0, 0), (2,0), and (2,2) in the xy-plane.

11. (7 points) Use Green's Theorem exactly once to prove the area of the region inside an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab.

12. (8 points) Use cylindrical coordinates to find the volume of the solid E that is the intersection of the solid parabola $z \geq x^2 + y^2$ and the solid ball $x^2 + y^2 + z^2 \leq 2$.

13. (6 points) Use spherical coordinates to evaluate $I = \displaystyle\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \int_0^{\sqrt{4-x^2-y^2}} \frac{1}{x^2 + y^2 + z^2}\, dz\, dx\, dy.$