Test 2 100 points Math 200 Name: ____________________

1. For all parts of this problem let $\vec{F}(x,y,z) = (6x^2)\hat{\imath} + (z^3 - \sin(y))\hat{\jmath} + (myz^2 - \cos(z))\hat{k}$.

   (a) (4 points) Find m so that $\vec{F}(x,y,z)$ is a gradient field.

   (b) (10 points) Use the FTCLI to evaluate $I = \displaystyle\int_C \vec{F}(x,y,z) \cdot d\vec{s}$ if C is parameterized by $\vec{\alpha}(t) = \langle t\sin(t), t - 2\pi, t\cos(t)\rangle$ for $0 \le t \le 2\pi$.

2. (6 points) Evaluate $I = \displaystyle\int_0^1 \int_0^x \int_0^{xy} \frac{4ze^{xy}}{y^2}\, dz\, dy\, dx$.

3. For all parts of this problem, $f(x, y)$ is a differentiable function with $f(3, -1) = 6$, $\nabla f(3, -1) = 4\hat{\imath} - 2\hat{\jmath}$, and $P = (3, -1, 6)$.

   (a) (4 points) Find the equation of the tangent plane in standard form to the graph of $z = f(x, y)$ at the point P.

   (b) (4 points) Use the linear approximation for $f(x, y)$ at the point P to estimate $f(2.9, -0.9)$.

   (c) (3 points) Find the minimum value of $D_{\vec{u}}f(3, -1)$. Show some work.

4. (6 points) Let $f(x, y) = x^2 - 3y$, $x(u, y) = 4u + y^2$, $y = y(u)$, $y(3) = 2$, and $y'(3) = -1$. Find $\left.\dfrac{df}{du}\right|_{u=3}$. Show organized work.

5. (6 points) Suppose $f(L, R) = \dfrac{L^2}{R^4}$ where L is measured with a 3% maximum percent error and R is measured with a 2% maximum percentage error. Use differentials to estimate the maximum percentage error of $f(L, R)$ when $L = 5$ and $R = 1$.

6. (6 points) Find the tangent plane in standard form for the graph of $x + 3y + 2z = 2\ln(x) + \ln(y) + \ln(z) + 6$ at the point $(1, 1, 1)$.

7. (6 points) Sketch the region of integration for $I = \displaystyle\int_0^2 \int_{-y}^{y} f(x, y)\, dx\, dy$ and then rewrite I with the order of integration switched to $dy\, dx$.

8. (8 points) Find the net volume for $f(x, y) = 5 - x^3 - yx$ over the region of integration bounded by $y = x^2$ and $y = 2 - x^2$.

9. (8 points) Find the mass of the part of the solid sphere $x^2 + y^2 + z^2 \leq 9$ for which $x \geq 0$, $y \geq 0$, and $z \geq 0$ if the density is $\delta(x, y, z) = 4x \quad \mathrm{kg/m^3}$.

10. (8 points) Use Green's Theorem exactly once to evaluate $I = \int_C y - \cos(y)\, dx + x\sin(y)\, dy$ if C is the circle $(x-3)^2 + (y+2)^2 = 16$ oriented clockwise.

11. (7 points) Use Green's Theorem to calculate the area of the region inside the closed curve parameterized by $\vec{\alpha}(t) = \langle 2\cos(t) + \sin(t), \sin(t)\rangle$ for $0 \le t \le 2\pi$.

12. (8 points) Use spherical coordinates to find the mass of the solid bounded by $z = \sqrt{x^2 + y^2}$ and $z = \sqrt{2 - x^2 - y^2}$ if the density is $\delta(x, y, z) = 3 \arctan\left(\frac{y}{x}\right)$.

13. (6 points) Use cylindrical coordinates to evaluate $I = \int_{-3}^{3} \int_{0}^{\sqrt{9-x^2}} \int_{0}^{9-x^2-y^2} \frac{1}{\sqrt{x^2 + y^2}} \, dz \, dy \, dx$.