Test #2 100 points Math 200 Name:____

Show work and always simplify your answers.

1) Find $\frac{\partial g}{\partial t}$ and $\frac{\partial g}{\partial w}$ in terms of t, w, g_x, and g_y if $g = g(x, y)$, $x = we^{2t}$, and $y = \frac{w}{t}$. (6 points)

2) Find the directional derivative of $f(x, y, z) = xy + z^3$ at $P = (-2, 2, -1)$ in the direction pointing towards the origin. (6 points)

3) Find an equation for the tangent plane in standard form to the surface $xy = -5 - z^3$ at $P = (-2, 2, -1)$.
(6 points)

4) Suppose $f(2,4)=4$ and $\nabla f(2,4)=\langle 5,-1\rangle$. Use differentials to estimate $f(1.8,4.3)$. (5 points)

5) Evaluate $\int_C \vec{F}\cdot\overrightarrow{dr}$ given the $\vec{F}$ and C below. Use the **FTCLI once** and **Green's Theorem once.**

a) $\vec{F}=\langle e^x yz, e^x z+2yz, e^x y+y^2\rangle$ and C is the curve $\vec{r}(t)=\langle 1+\cos t, 1+\sin t, t\rangle$ for $0\le t\le\pi$. (6 points)

b) $\vec{F}=\langle y, xy\rangle$ and C is the closed curve that starts at (1, 0) and rotates about the ellipse $4x^2+y^2=4$ counterclockwise in the xy-plane. (6 points)

6) Use a Riemann sum to estimate $\iint_R x - y\,dA$ using midpoints and a partition with two rows and two columns if R is the rectangle with vertices [2, 10] x [1, 5]. Draw a picture and indicate the points used to evaluate the integrand. (5 points)

7) Sketch the region of integration for I = $\int_0^2 \int_x^{2x} f(x, y)\,dydx$ and then write I as an integral expression with the order of integration switched. (8 points)

8) **Use a double integral** to find the volume of the solid bounded by the coordinate planes and by $x + y + z = 1$. (6 points)

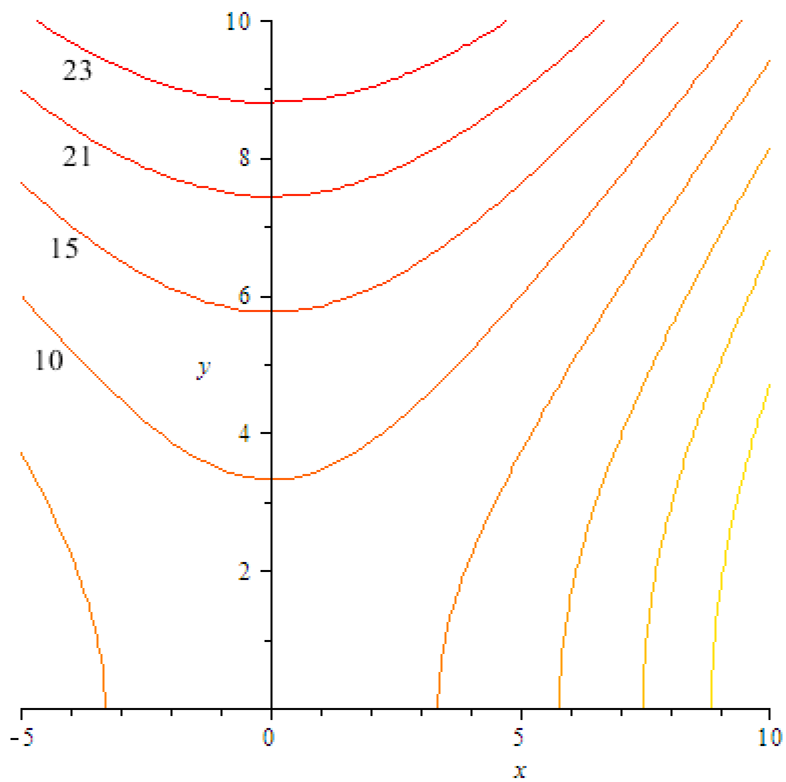
9) **Use polar coordinates** to evaluate $I = \int_0^4 \int_0^{\sqrt{16-x^2}} \tan^{-1}\left(\frac{y}{x}\right) dydx$. (8 points)

10a) Find the curl and divergence of $\vec{F} = \langle xy, xyz, y^2 \rangle$. (6 points)

10b) Use your answer to 10a) to **estimate** the work done by $\vec{F} = \langle xy, xyz, y^2 \rangle$ on a particle that moves about a closed curve with very small diameter in the plane 2x + 2y + z = 5 if the curve contains the point (1, 1, 1) inside it and the region inside the curve has area 0.1. (1 point)

11) Find the center of mass of the lamina in the shape of a quarter circle; it is bounded by $x^2 + y^2 = 4$ in the first quadrant (x and y are both nonnegative.) The density of the solid is the constant $\rho(x, y) = 2$ grams per cm^2. (8 points)

12) The level curves of $f(x, y)$ are shown below. Sketch the gradient vector $\nabla f(2,6)$. You may assume that the point (2, 6) lies on the level curve z = 15. (7 points)



13) **Write Green's Theorem** and then **verify it** if R is the region inside the circle r = 2, and if $\vec{F} = \langle x, x + 2xy \rangle$.

Green's Theorem:____(4 points)

Verify.

Calculate the quantity on the left side of the Theorem as you stated it. (6 points)

Calculate the quantity on the right side of the Theorem as you stated it. (6 points)