Test #2 100 points Math 200 Name:______________________________________

Show work and always simplify your answers.

1) Find $\frac{\partial g}{\partial t}$ at t = 2 and w = 3 if $g(x,y) = \cos(\text{xy})$, $x = x(t,w)$, with $x(2,3) = 1$, $x_t(2,3) = 2$, and $y = \frac{\pi t w}{4}$.
(6 points)

2) Find the directional derivative of $f(x,y) = x^2 y + xy^2$ at $P = (-2,2)$ in the direction towards the point $(1,-2)$.
(6 points)

3) Find an equation (in standard form) for the tangent plane to the surface $e^{x+y+z} = 4x + 2y + 3z + 1$ at $P = (0,0,0)$. (6 points)

4) Suppose $f(2,4)=4$ and $\nabla f(2,4)=\langle 5,-1\rangle$. Use differentials to estimate $f(1.8,4.3)$. (5 points)

5) Suppose $f(2,4)=4$ and $\nabla f(2,4)=\langle 5,-1\rangle$. Find the tangent plane to the graph of z = f(x, y) at the point (2, 4, 4). Write your final answer in ax +by +cz = d form. (5 points)

6) Use the **FTCLI once** to evaluate $\int_C \vec{F}\cdot d\vec{s}$ if $\vec{F}=\langle e^x y, e^x+2y\rangle$ and C is the curve $\vec{r}(t)=\langle 1+\cos t, 1+\sin t\rangle$ for $0\le t\le \pi$. (6 points)

7) Use **Green's Theorem once** to evaluate $\int_C \vec{F} \cdot d\vec{s}$ if $\vec{F} = \langle xy, y \rangle$ and C is the closed curve that rotates once counterclockwise about the triangle with vertices (2, 0), (2, 4), and (0, 4). (6 points)

8) Sketch the region of integration for I = $\int_0^2 \int_{-2x}^{x} f(x, y)\, dydx$ and then write I as an integral expression with the order of integration switched. (6 points)

9) Evaluate $\int_0^1 \int_{-z}^{z} \int_0^y y^2 + 2zx\, dxdydz$. (6 points)

10) **Use a double integral** to find the volume of the solid bounded by $z = 2 - x^2 - y^2$ and $z = -6 + x^2 + y^2$. (6 points)

11) Use polar coordinates to evaluate $I = \int_0^4 \int_0^{\sqrt{16-x^2}} \tan^{-1}\left(\frac{y}{x}\right) dydx$. (6 points)

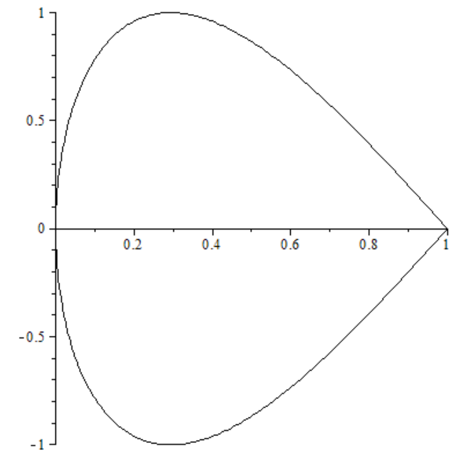
12) Find any two numbers a and b so that $\vec{F} = \langle a\sin(2y) + y^2, 8\cos(2y) + byx \rangle$ is a gradient field. Defend your answer. (6 points)

13) Find $\bar{y}$, the y-coordinate of the center of mass of the lamina that is bounded by $1 \le x^2 + y^2 \le 4$ in the first quadrant (x and y are both nonnegative.) The density of the lamina is $\delta(x, y) = x$ grams per cm^2. (6 points)

14) Find I_0, the moment of inertia about (0, 0), for the lamina that is bounded by $1 \le x^2 + y^2 \le 4$ in the first quadrant (x and y are both nonnegative.) The density of the lamina is $\delta(x, y) = x$ grams per cm^2. (6 points)

15) Suppose $f(m,v) = \frac{1}{2}mv^2$ where m is measured with a 10% maximum percent error and v is measured with a 4% maximum percentage error. Use differentials to estimate the maximum percentage error of f(m, v) when m = 10 and v = 2. (6 points)

16) **Use Green's Theorem** to find the area of the region bounded by the curve parameterized by $\vec{r}(t) = \langle 1+\cos(t), \sin(2t) \rangle$, $\frac{\pi}{2} \le t \le \frac{3\pi}{2}$. (6 points)



17) The level curves of $f(x, y)$ are shown below so that the top level curve is f(x, y) = 100 and all the other level curves represent outputs two units smaller than those of the level curve above it. Sketch the gradient vector $\nabla f(2,0)$ after estimating its magnitude. (6 points)