Test 2 100 points Math 200 Name: ______________________

1. (6 points) Sketch the region of integration for $I = \int_0^4 \int_{-x}^{x^2/4} f(x,y)\,dy\,dx$ and then rewrite I with the order of integration switched.

2. (7 points) Find the tangent plane in standard form for the graph of $ye^x + \sin(y)\cos(z) = z$ at the point $Q = (0, \pi, \pi)$.

3. (5 points) Approximate $f(3.2, 0.9)$ using a differential if $f(3,1) = 2$ and $\nabla f(3,1) = \langle 5, -2 \rangle$.

4. (4 points) If $\vec{F}(x, y) = (mx^2y)\hat{\imath} + (x^3 - \cos(y))\hat{\jmath}$ then $\vec{F}$ is a gradient field if $m = 3$ and is not a gradient field if $m \neq 3$. Explain why this is true. Give a complete explanation.

5. (7 points) Use the FTCLI to evaluate $I = \displaystyle\int_C 3x^2y\,dx + x^3 - \cos(y)\,dy$ if C is parameterized by $\vec{\alpha}(t) = \langle t\cos(t), t - 2\pi\rangle$ for $0 \leq t \leq \pi$.

6. (7 points) Evaluate $I = \displaystyle\int_C x^2y\,dx + x^3 - \cos(y)\,dy$ if C is parameterized by $\vec{\alpha}(t) = \left\langle t^2, t\right\rangle$ for $-\pi \leq t \leq \pi$.

7. (10 points) Evaluate $I = \int_{-1}^{1} \int_{0}^{y} \int_{y-x}^{x-y} 2z + 3 \, dz \, dx \, dy$.

8. (4 points) What is the tangent plane to the surface $z = f(x, y)$ at the point $P = (3, 1, 2)$ if $f(3, 1) = 2$ and $\nabla f(3, 1) = \langle 4, 5 \rangle$? Write your final answer in $ax + by + cz = d$ form.

9. (4 points) Use Green's Theorem and a double integral to evaluate $I = \int_C \langle \cos(x^3) - x^6, 2x + e^{8y} \rangle \cdot d\vec{s}$ if C is the closed curve that starts at $(2, 0)$ travels on the x-axis to $(-2, 0)$ and then back to $(2, 0)$ along the curve $y = \sqrt{4 - x^2}$.

10. (8 points) Let $f(x, y) = x^2 \cos(y)$, $x(u, w) = \dfrac{4u + w^2}{u + 1}$, $y = y(w)$, $y(1) = \dfrac{\pi}{3}$, and $y'(1) = -\dfrac{2}{\sqrt{3}}$.

Find $\dfrac{df}{dw}$ when $w = 1$ and $u = 0$. Show organized work.

11. (4 points) Find the directional derivative of $f(x, y) = x\ln(x) - x\cos(2y)$ at the point $P = (e, \pi)$ in the direction of $\langle 1, 1 \rangle$.

12. (6 points) Find the moment of inertia about the origin, I_0, for the lamina R equal to the half disk $0 \le x \le \sqrt{1 - y^2}$ in the xy-plane if its density is $\delta(x, y) = \dfrac{x}{x^2 + y^2}$.

13. (7 points) Find the volume of the solid bounded by $z = x^2 + y^2$ and $z = 3x^2 + 3y^2 - 8$.

14. (7 points) Use Green's Theorem to find the area of the region R bounded by the closed curve C parameterized by $\vec{p}(t) = \langle 5\cos(t) - \cos(5t), 5\sin(t) + 0.2\sin(5t)\rangle$ for $0 \leq t \leq 2\pi$.

15. let $I = \int_{-3}^{3} \int_{0}^{\sqrt{9-x^2}} \int_{0}^{9-x^2-y^2} \frac{1}{\sqrt{x^2+y^2}} \, dz \, dy \, dx$ for both parts below.

(a) (4 points) Write I using spherical coordinates. **Do not evaluate or simplify!**

(b) (4 points) Write I using cylindrical coordinates. **Do not evaluate or simplify!**

16. (6 points) The lamina R is equal to the half disk $0 \leq x \leq \sqrt{1-y^2}$ in the xy-plane with density $\delta(x, y) = \frac{x}{x^2+y^2}$. Find the center of mass. Hint: $\bar{y} = 0$ by symmetry so just find $\bar{x}$.