Test #3A 100 points Math 200 Name:____

Simplify your final answers. Show organized work. Defend all answers.

1) Use spherical coordinates to evaluate I = $\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} \frac{1}{x^2+y^2+z^2}\, dzdydx$. (10 points)

2) Use cylindrical coordinates to find the volume of the solid that lies between $z = -x^2 - y^2$ and $x^2 + y^2 + z^2 = 2$. (10 points)

3) Convert $(-1, 0, -\sqrt{3})$ to spherical coordinates.

4) Use **Green's** Theorem to evaluate I = $\int_C \sin x\, dx + x^2 y^3\, dy$ if C is the closed curve that rotates once **clockwise** about the triangle with vertices (0, 0), (2, 0), and (2, 2) in the xy-plane. (10 points)

5) Use **Green's** Theorem to find the area of the region between the x – axis and one arch of the cycloid parameterized by $\mathbf{p}(t) = \langle t - 2\sin(t), 2 - 2\cos(t)\rangle$ for $0 \le t \le 2\pi$. (10 points)

6) Let S be the piece of $\rho = 3$ that is below $\phi = \dfrac{\pi}{6}$ and is oriented outward. Write one $d\vec{S}$ for all of S. (5 points)

7) Find the mass of the surface S parameterized by $\mathbf{p}(u,v)=\langle u+v,2-3u,1+u-v\rangle$, $-1\le u\le 1$ and $0\le v\le 2$ if the density is $\delta(x,y,z)=x+y+z$ grams per square meter. (15 points)

8) Find the flux of $F=\langle -x,-y,z\rangle$ through the part of $z=\sqrt{x^2+y^2}$, **oriented down**, that lies between the planes z = 1 and z = 4. (10 points)

9) Evaluate I = $\iint_R (x+y)e^{x^2-y^2}\,dA$ by making an appropriate substitution, where R is the parallelogram enclosed by the lines $x-y=0$, $x-y=2$, $x+y=0$, and $x+y=3$. (10 points)

10) R is the region in the **upper xy - plane** bounded by $y=|x|$ and the circles r = 1 and r = 2. Use this region for both parts below. (15 points)

10A) Find the y – coordinate of the center of mass for the region R if the density is $\delta(x,y)=\dfrac{y}{x^2+y^2}$.

10B) Find the moment of inertia, I_0, for R with the same density, $\delta(x,y)=\dfrac{y}{x^2+y^2}$, as in #10A.