

**Simplify your final answers. Show organized work. Defend all answers.**

1) Use spherical coordinates to evaluate  $I = \int_0^1 \int_0^{\sqrt{1-x^2}} \int_{-\sqrt{2-x^2-y^2}}^{-\sqrt{x^2+y^2}} \frac{1}{x^2 + y^2 + z^2} dz dy dx$ . (10 points)

2) Use cylindrical coordinates to find the volume of the solid that lies between  $z = x^2 + y^2$  and  $x^2 + y^2 + z^2 = 2$ . (10 points)

3) Convert  $(1, 1, -\sqrt{2})$  to spherical coordinates.

4) Use **Green's** Theorem to evaluate  $I = \int_C x^2 y^3 dx + \sin(y) dy$  if  $C$  is the closed curve that rotates once **clockwise** about the triangle with vertices  $(0, 0)$ ,  $(2, 0)$ , and  $(2, 2)$  in the  $xy$ -plane. (10 points)

5) Use **Green's** Theorem to find the area of the region between the  $x$  – axis and one arch of the cycloid parameterized by  $\mathbf{p}(t) = \langle t - 3\sin(t), 3 - 3\cos(t) \rangle$  for  $0 \leq t \leq 2\pi$ . (10 points)

6) Let  $S$  be the piece of  $\rho = 3$  that is below  $\phi = \frac{\pi}{3}$  and is oriented outward. Write one  $d\vec{S}$  for all of  $S$ . (5 points)

7) Find the mass of the surface  $S$  parameterized by  $\mathbf{p}(u, v) = \langle 2 - 3u, u + v, 1 + u - v \rangle$ ,  $-1 \leq u \leq 1$  and  $0 \leq v \leq 2$  if the density is  $\delta(x, y, z) = x + y + z$  grams per square meter. (15 points)

8) Find the flux of  $F = \langle x, -y, -z \rangle$  through the part of  $z = \sqrt{x^2 + y^2}$ , **oriented down**, that lies between the planes  $z = 1$  and  $z = 4$ . (10 points)

9) Evaluate  $I = \iint_R (x-y)e^{x^2-y^2} dA$  by making an appropriate substitution, where R is the parallelogram enclosed by the lines  $x-y=0$ ,  $x-y=2$ ,  $x+y=0$ , and  $x+y=3$ . (10 points)

10) R is the region in the **upper xy - plane** bounded by  $y=|x|$  and the circles  $r=1$  and  $r=2$ . Use this region for both parts below. (15 points)

10A) Find the y - coordinate of the center of mass for the region R if the density is  $\delta(x, y) = \frac{y}{x^2 + y^2}$ .

10B) Find the moment of inertia,  $I_0$ , for R with the same density,  $\delta(x, y) = \frac{y}{x^2 + y^2}$ , as in #10A.