Test #3B 100 points Math 200 Name:________________________________

Simplify your final answers. Show organized work. Defend all answers.

1) Use either cylindrical or spherical coordinates to evaluate the following integrals. (20 points)

A) I = $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{0}\int_{0}^{\sqrt{4-x^2-y^2}} e^{-(x^2+y^2+z^2)^{3/2}}\, dzdydx$.

B) I = $\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}}\int_{0}^{\sqrt{x^2+y^2}} z\, dzdydx$.

2) Use **Green's** Theorem to evaluate $\int_C \vec{F}\cdot d\vec{r}$ if $\vec{F} = \langle 5x^2, 3xy\rangle$ and C is the closed curve that starts at (0, 0) and rotates once **clockwise** about the rectangle [0, 4] x [0, 2] in the xy-plane. (10 points)

3) Find the mass of the piece of the sphere $\rho = 2$ that lies **below** the cone $\phi = \frac{3\pi}{4}$ if the density is $\delta = \frac{1}{x^2 + y^2 + z^2}$ grams per square centimeter. (10 points)

4) Find the curl and divergence of $\vec{F} = \left\langle xz, zy, x^2 y \right\rangle$. Label each answer with appropriate notation. (10 points)

5) Find the flux of $\vec{F} = \langle 1 - x, y - x + 1, z \rangle$ through the surface parameterized by $\mathbf{p}(u,v) = \langle 1 - u, v - u, uv \rangle$, **oriented up**, if $0 \le u \le 1$ and $0 \le v \le 6$. (10 points)

6) R is the square in the xy - plane with vertices $(\pi,0)$, $(\pi,2\pi)$, $(0,\pi)$, and $(2\pi,\pi)$. Change variables to find I = $\iint_R \left((x+y)\cos(x-y)\right)^2 dxdy$. (10 points)

7) Find the flux of $\vec{F} = \langle y, x, z \rangle$ through the part of the plane $x+y+z=4$, **oriented down**, that lies inside the cylinder $x^2+y^2=4$. (10 points)

8) Find the mass of the piece of the cylinder $x^2 + y^2 = 1$ that lies above z = 0 and below $z = 1 - x$ if the density is $\delta = 1 + x$ grams per square centimeter. (10 points)

9) Use **Green's** Theorem to find the area of the region inside the closed curve C parameterized by $\mathbf{r}(t) = \langle 2 + \sin(t), 1 - \sin(2t) \rangle$ for $0 \le t \le \pi$. The graph of C is shown below. (10 points)