Test #3C 100 points Math 200 Name:____

Simplify your final answers. Show organized work. Defend all answers.

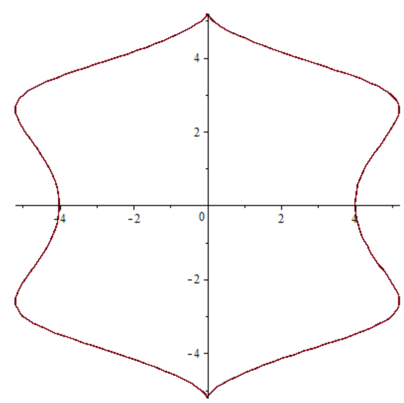
1) Use spherical coordinates to evaluate I = $\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} \frac{1}{x^2+y^2+z^2}\, dzdydx$. (10 points)

2) Use cylindrical coordinates to find the volume of the solid that lies between $y = x^2 + z^2$ and $y = 8 - x^2 - z^2$. (10 points)

3) Convert $(-1, 1, \sqrt{2})$ to spherical coordinates. (5 points)

4) Use **Green's** Theorem to evaluate I = $\int_C \sin(y)\,dx + x^2 y^3\,dy$ if C is the closed curve that rotates once **clockwise** about the triangle with vertices (0, 0), (0, 2), and (2, 2) in the xy-plane. (10 points)

5) Use **Green's** Theorem to find the area of the region shown below if the boundary is parameterized by $\mathbf{p}(t) = \langle 5\cos(t) - \cos(5t), 5\sin(t) + (0.2)\sin(5t) \rangle$ for $0 \le t \le 2\pi$. (10 points)



6) Let S be the piece of $\rho = 3$ that is above $\phi = \dfrac{3\pi}{4}$ and is oriented outward. Write one $d\vec{S}$ for all of S. (5 points)

7) Find the mass of the surface S parameterized by $\mathbf{p}(u,v) = \langle 3-2u, u-v, 1+u-v \rangle$, $0 \le u \le 3$ and $-2 \le v \le 2$ if the density is $\delta(x,y,z) = x+y+z$ grams per square meter. (15 points)

8) Find the flux of $F = \langle x, -y, -z \rangle$ through the part of $z = x^2 + y^2$, **oriented down**, that lies between the planes z = 1 and z = 4. (10 points)

9) Evaluate I = $\iint_R (x-y)e^{x^2-y^2}\,dA$ by making an appropriate substitution, where R is the parallelogram enclosed by the lines $x-y=0$, $x-y=2$, $x+y=0$, and $x+y=3$. (10 points)

10) R is the part of $r \le 2$ in the xy – plane for which $x \ge 0$. Use this region for both parts below.

10A) Find the x – coordinate of the center of mass for the region R if the density is $\delta(x,y) = \dfrac{x}{x^2+y^2}$.

(10 points)

10B) Find the moment of inertia, I_0, for R with the same density, $\delta(x,y) = \dfrac{x}{x^2+y^2}$, as in #10A. (5 points)