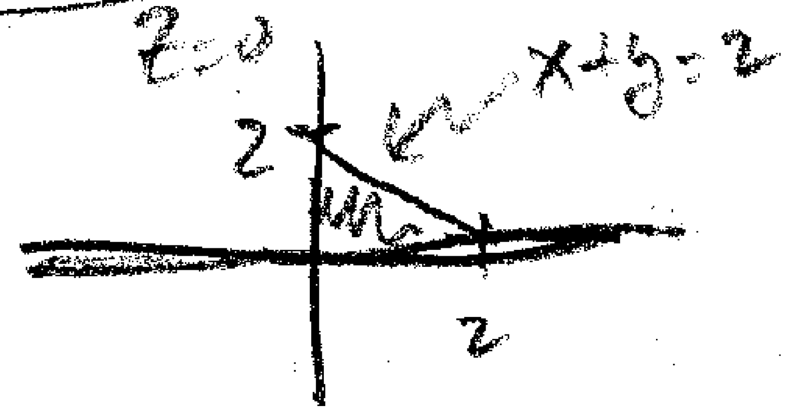
Math 200 Test #3 Answers

(1) Inner $= (4\sin(y) + xy)\Big|_x^{4x} = 4(\sin(4x) - \sin(x)) + 4x^2 - x^2$

$$\Rightarrow I = \int_{-\pi}^{\pi} 4(\cancel{\sin(4x)}^{0} - \cancel{\sin(x)}^{0}) + 3x^2\,dx = 2\cdot x^3\Big|_0^{\pi} = \boxed{2\pi^3}$$

(2) 

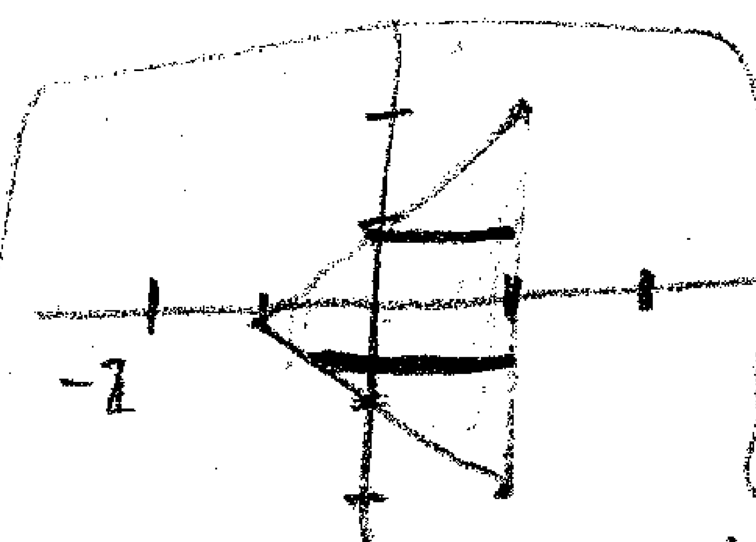
$$\Rightarrow Vol = \int_0^2\int_0^{2-x} \frac{2-x-y}{2}\,dy\,dx = \int_0^2 \underbrace{(2-x) - \frac{x(2-x)}{2} - \frac{(2-x)^2}{4}}\,dx =$$

$$= (2-x)\left(1 - \frac{x}{2} - \frac{(2-x)}{4}\right)$$

$$= (2-x)\left(\frac{2}{4} - \frac{x}{4}\right) = \frac{1}{4}(2-x)^2$$

$$\frac{1}{4}\int_0^2 (2-x)^2\,dx = \frac{1}{4}\,\frac{(2-x)^3}{-3}\Big|_0^2 = \frac{1}{4}\cdot\frac{8}{3} = \boxed{\frac{2}{3}}$$

(3) $-1 \le x \le 1$

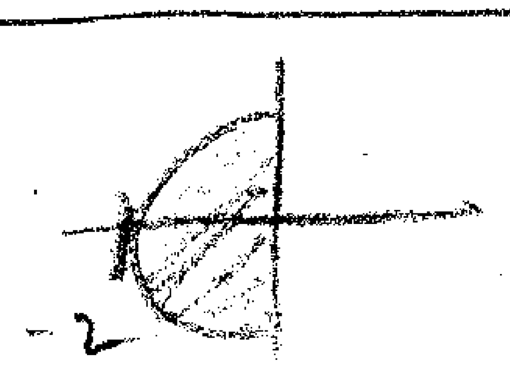
$-x-1 \le y \le x+1 \Rightarrow$ 

$\Rightarrow -y-1 \le x \le 1$ with $-2 \le y \le 0$ and $0 \le y \le 2$, $y-1 \le x \le 1$, so

$$I = \int_{-2}^{0}\int_{-y-1}^{1} f(x,y)\,dx\,dy + \int_0^2\int_{y-1}^{1} f(x,y)\,dx\,dy$$

(4) $\text{curl}\,\vec F = \nabla\times\vec F = \boxed{\langle -1, -1, -x\rangle}$ | $\text{Div}\,\vec F = \nabla\cdot\vec F = y + 0 + 0 = \boxed{y}$

(5) $$\int_C \vec F\cdot d\vec r \underset{\text{Green's Thm}}{=} \iint_R 1+1\,dA = 2(\text{Area of ellipse}) = 2\cdot\pi\cdot1\cdot2 = \boxed{4\pi}$$

(6) 

$$\Rightarrow I = \int_{\pi/2}^{3\pi/2}\int_0^2 \theta\, r\,dr\,d\theta = \frac{\theta^2}{2}\Big|_{\pi/2}^{3\pi/2}\cdot\frac{r^2}{2}\Big|_0^2 = \frac{1}{2}\left(\frac{9\pi^2}{4} - \frac{\pi^2}{4}\right)\cdot 2$$

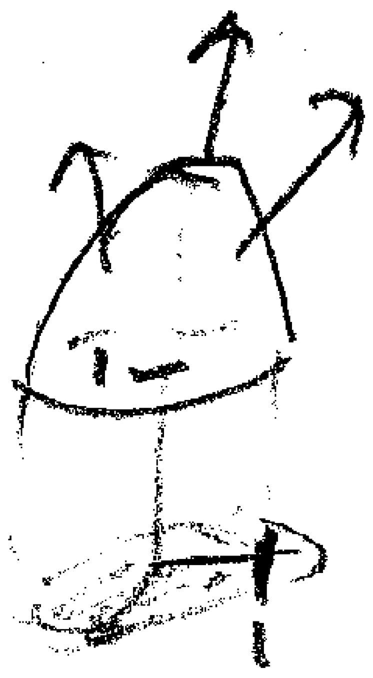
$$= \boxed{2\pi^2}$$

(7) $\bar{x} = \frac{M_y}{M}$. $M = \int_0^1\int_0^2 xy\,dy\,dx = \frac{x^2}{2}\Big|_0^1\cdot\frac{y^2}{2}\Big|_0^2 = 1.$

$$M_y = \int_0^1\int_0^2 x^2 y\,dy\,dx = \frac{x^3}{3}\Big|_0^1\cdot\frac{y^2}{2}\Big|_0^2 = \frac{2}{3}.$$

So $\boxed{\bar{x} = 2/3}$

(8)



$\Rightarrow d\vec{S} = \langle 2x, 2y, 1\rangle\, dx\, dy$, so $\vec{F}\cdot d\vec{S} = 2xy - 2xy + z$

$\Rightarrow \text{Flux} = \iint\limits_{x^2+y^2\le 1} 2 - x^2 - y^2 \, dA = \int_0^{2\pi}\int_0^1 2r - r^3 \, dr\, d\theta$

$= r^2 - \frac{r^4}{4}\Big|_0^1 \cdot 2\pi = \boxed{\frac{3\pi}{2}}$

(9)

$z = 4 - x - y \Rightarrow dS = |\langle 1, 1, 1\rangle|\, dx\, dy = \sqrt{3}\, dx\, dy$, so

$\text{Mass} = \iint\limits_{x^2+y^2\le 1} x^2\sqrt{3}\, dA = \sqrt{3}\int_0^{2\pi}\int_0^1 r^2\cos^2\theta \cdot r\, dr\, d\theta$

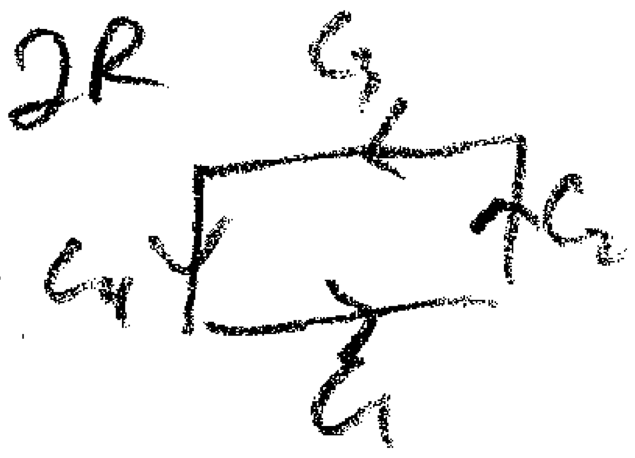
$= \sqrt{3}\cdot \frac{r^4}{4}\Big|_0^1 \cdot 2\pi \cdot \frac{1}{2} = \boxed{\frac{\sqrt{3}}{4}\pi \text{ grams}}$

(10)

$\iint\limits_R \frac{\partial(x^2)}{\partial x} - \frac{\partial(xy)}{\partial y}\, dA = \int_0^3\int_0^1 x\, dy\, dx = \frac{x^2}{2}\Big|_0^3 \cdot 1 = \boxed{\frac{9}{2}}$

Green's theorem is verified

$\oint\limits_{\partial R} \vec{F}\cdot d\vec{r} = \underset{y=0}{\int_0^3 0\, dx} + \underset{x=3}{\int_0^1 9\, dy} + \underset{y=1}{\int_3^0 x\, dx} + \underset{x=0}{\int_1^0 0\, dy}$



$= 0 + 9 - \frac{x^2}{2}\Big|_0^3 + 0 = 9 - \frac{9}{2} = \boxed{\frac{9}{2}}$ ✓