

Simplify your final answers. Show organized work. Defend all answers.

1) Find the center of mass for the lamina that is the part of $1 \leq x^2 + y^2 \leq 4$ **in the first quadrant** of the xy – plane if its density equals $\frac{1}{\sqrt{x^2 + y^2}}$ grams per square meter. (10 points)

2) Find the moment of inertia about the origin, I_0 , for the lamina and density in #1. (5 points)

3) Use Green's Theorem to find the work done by $\vec{F} = \langle y, x^2 \rangle$ on a particle that moves once counterclockwise around the boundary of the lamina from #1. (10 points)

4) Find the volume of the solid that is inside the sphere $\rho = 2$, but **outside** the cone $z = r$. (10 points)

5) Find the mass of the solid bounded by the cylinder $r = 2$, the plane $z = -1$, and the plane $x + y + z = 5$ if the density is $\delta(x, y, z) = \sqrt{x^2 + y^2}$ grams per square cm. (10 points)

6) Use **Green's Theorem** to find the area of the region inside the closed curve C parameterized by $\vec{r}(t) = \langle \cos t + \sin t, \sin t \rangle$, $0 \leq t \leq 2\pi$. (5 points)

- 7) Find the area of the surface parameterized by $\vec{\alpha}(u, p) = \langle \cos u, \sin u, \sin p \rangle$ for $0 \leq u \leq \frac{\pi}{2}$, and $0 \leq p \leq \frac{\pi}{2}$.
(13 points)

- 8) Find the flux of $\vec{F}(x, y, z) = y\hat{\mathbf{j}} - z\hat{\mathbf{k}}$ through the part of $z = xe^y$ with $0 \leq x \leq 1$ and $0 \leq y \leq 1$, oriented up.
(12 points)

9) Find the flux of $\vec{F}(x, y, z) = x\hat{\mathbf{i}} - z\hat{\mathbf{j}} + y\hat{\mathbf{k}}$ through the part of $x^2 + y^2 + z^2 = 4$ that lies in the first octant oriented **towards the origin**. (13 points)

9) Find the mass of the part of $y^2 + z^2 = 4$ that lies between $x = 0$ and $x = 3$ **in the first octant** if the density is $\delta(x, y, z) = z + x^2y$ grams/cm². (12 points)