Test #3 100 points Math 200 Name:______________________________________

Show work.

1) Find and classify all the critical points of $f(x,y)=3y+\left(x^2-4x\right)\ln(y)$. (10 points)

2) Find and classify the extreme values of $f(x,y)=2x^2+8y^2$ for all points that lie on $xy=8$. **Sketch** a picture of level curves to verify your answer. (10 points)

3) Use the divergence theorem to find the flux of $\vec{F}(x,y,z)=\langle x^2-y, z^2, 3z+x\rangle$ out of the cube with side length **four** that is centered at (0, 0, 0). (10 points)

4) Use Stokes' theorem to find the flux of $\nabla\times\vec{F}$ through the hemisphere $z=\sqrt{1-x^2-y^2}$ that is oriented with nonnegative z-coordinates if $\vec{F}(x,y,z)=\langle x^2-y, z^2, 3z+x\rangle$. Note: there is no bottom to the hemisphere.
(10 points)

5) Use a substitution to find I = $\iint\limits_R \cos(x+y)$ dA if R is the region in the xy – plane bounded by $x+y=\frac{\pi}{2}$, $x+y=\pi$, $x-y=0$, and $x-y=3$. (10 points)

6) Find I = $\iint\limits_S \vec{F}(x,y,z)\cdot d\vec{S}$ if S is oriented up and parameterized by $\vec{r}(u,\theta)=\left\langle u\cos\theta, u\sin\theta, \sin\left(u^2\right)\right\rangle$, $\sqrt{\frac{\pi}{2}}\le u\le\sqrt{\pi}$, $\frac{\pi}{6}\le\theta\le\frac{\pi}{3}$, and $\vec{F}(x,y,z)=\left\langle \frac{2}{x},\frac{2}{y},0\right\rangle$. (10 points)

7) Find I = $\iint_S \vec{F}(x,y,z)\cdot d\vec{S}$ if S is the piece of $z = y^2 + 5$, oriented up, such that $1 \le x^2 + y^2 \le 9$, and $\vec{F}(x,y,z) = \langle 0, -y, z \rangle$. (10 points)

8) Calculate I = $\iint_S z\,dS$ if S is the piece of $x^2 + y^2 = 4$ in the **first octant** that lies below the plane z = y. (10 points)

9) Find the mass of the part of the plane $2x+3y+2z=6$ that lies in the first octant if the density is $\delta(x,y,z)=y$ grams per square meter. (10 points)

10) Use the definition of divergence to estimate the flux of $\vec{F}(x,y,z)=\left\langle 2x+3y, z^2-7y^2, z-4x^2\right\rangle$ out of the cube centered at (0, 0, 0) with side length 0.01 centimeters. (4 points).

11) Use the divergence theorem to find I = $\iint_S 3x^2 + 2y^2 + z \, dS$ if S is the sphere $x^2 + y^2 + z^2 = 9$, oriented outward. (4 points)

12) Use the definition of curl to estimate the work done by $\vec{F}(x, y, z) = \langle 2x + 3y, z^2 - 7y^2, z - 4x^2 \rangle$ on a particle that moves once around the curve C parameterized by $\vec{r}(t) = 0.01\langle \cos(t), \sin(\mathrm{t}), 0 \rangle$. (2 points)