Test 3 100 points Math 200 Name: ____________________

1. (9 points) Use a surface integral to find the mass of the sphere $\rho = 1$ if the density is $\delta(x, y, z) = x^2 + y^2$ grams/cm^2.

2. (9 points) Use a surface integral to find the flux of $\vec{F}(x, y, z) = \langle x, y, y \rangle$ through the graph of $z = 1 - x^2 + y$ oriented up if $-1 \leq x \leq 1$ and $-1 \leq y \leq 1$.

3. (9 points) Find the flux of $\vec{F}(x, y, z) = \langle z, 2, z \rangle$ through the surface parameterized by $\vec{p}(u, v) = \langle uv, u + v, 2v \rangle$ if $u^2 + v^2 \leq 1$ oriented with a positive $\hat{\imath}$ - component.

4. (9 points) Evaluate $I = \iint\limits_R \frac{y}{x}\, dx\, dy$ using a substitution if R is in the first quadrant bounded by $y = x$, $y = 3x$, $xy = 1$, and $xy = 3$.

5. (8 points) Use Lagrange Multipliers to find and classify the global extreme values of $f(x, y) = x + y$ constrained by $x^2 + 4y^2 = 20$. Draw a picture of level curves to verify your final answer.

6. (8 points) Find and classify all critical points for $f(x, y) = x^3 - 12xy + 8y^3$.

7. (8 points) Use the Divergence Theorem to find the flux of $\vec{F}(x,y,z) = \left\langle x^2 z, 4y^2, -5xz^2 \right\rangle$ out of the unit cube with x, y, and z between 0 and 1.

8. (8 points) Find the mass of the part of the cylinder $x^2 + y^2 = 4$ for which $x \geq 0$, $y \geq 0$, and $0 \leq z \leq 3$ if the density is $\delta(x,y,z) = z + x^2 y \quad kg/m^2$.

9. (8 points) Use Stokes Theorem **once** to find $I = \iint_S \nabla \times \vec{F} \cdot d\vec{S}$ if $\vec{F}(x, y, z) = \langle z, x, x^2 + y^2 \rangle$ and S is the **lower** hemisphere $\rho = 2$, $z \leq 0$, oriented down.

10. (8 points) Use the Divergence Theorem to find $I = \iint_{Bd(E)} 3x^2 + zy \, dS$ if E is the solid ball $x^2 + y^2 + z^2 \leq 4$.

11. (8 points) Use Stokes Theorem to find the work done by $\vec{F}(x, y, z) = \langle -y^2, -z^2, -x^2 \rangle$ if C is the triangle with vertices $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$ that is oriented counterclockwise from above.

12. (8 points) Prove that the volume of a solid E is equal to $\frac{1}{3} \iint\limits_{Bd(E)} \vec{F} \cdot d\vec{S}$ if $\vec{F}(x, y, z) = \langle x, y, z \rangle$.