Test 3 100 points Math 200 Name: ______________________________

1. (9 points) Use a surface integral to find the mass of the piece of the cylinder $x^2 + y^2 = 4$ bounded between $z = 0$ and $y + z = 4$ if the density is $\delta(x, y, z) = x^2$ grams/cm^2.

2. (9 points) Use a surface integral to find the flux of $\vec{F}(x, y, z) = \langle z, x, z \rangle$ through the piece of the graph of $z = x + y^2$ oriented up if it lies over the region in the xy-plane bounded by $y = x$, $y = 1$, and $x = 0$.

3. (9 points) Find the area of the surface parameterized by $\vec{p}(u,v) = \langle u - v, uv, 2\rangle$ if $u^2 + v^2 \leq 1$ **for nonnegative** u **and** v.

4. (9 points) Evaluate $I = \iint\limits_R \sin(9x^2 + 4y^2)\, dx\, dy$ using a substitution if R is the solid ellipse $9x^2 + 4y^2 \leq 1$. Hint: Substitute so that the ellipse becomes a circle.

5. (8 points) Use Lagrange Multipliers to find and classify the global extreme values of $f(x, y) = x^2 + 4y^2$ constrained by $x + y = 5$. Draw a picture of level curves to verify your final answer.

6. (8 points) Find and classify all critical points for $f(x, y) = \ln(xy^2) - x - 4y$. Be sure to determine if any extrema are local or absolute and defend your answer. You may use the fact that $\lim_{u \to \infty} \frac{u}{\ln(u)} = \infty$.

7. (8 points) Use Stokes' Theorem to find the work done by $\vec{F}(x,y,z) = \left\langle x^2z, 4y^2, -5xz^2 \right\rangle$ on a particle traveling about the square from $(0,0,0)$ to $(1,0,0)$ to $(1,0,1)$ to to $(0,0,1)$ and then back to the origin.

8. (8 points) Find the mass of the part of the sphere $x^2+y^2+z^2=4$ for which $x^2+y^2 \leq 1$ and $z \geq 0$ if the density is $\delta(x,y,z) = z \quad \text{kg/m}^2$.

9. (9 points) Use the Divergence Theorem **once** to find the flux of $\vec{F}(x, y, z) = \langle xz, 3y, x^2 + y^2 \rangle$ out of the boundary of the solid hemisphere, E, equal to $\rho \leq 2$ for $z \geq 0$.

10. (7 points) Use the Divergence Theorem to find $I = \iint_{Bd(E)} 3x^2 + zy\, dS$ if E is the solid ball $x^2 + y^2 + z^2 \leq 4$.

11. (8 points) Use Stokes Theorem **once** to find the flux of $\nabla \times \vec{F}$ through the graph of $z = 1 - x^2 - y^2$ for $z \geq 0$ oriented up if $\vec{F}(x, y, z) = \left\langle -y, -z^2, -x^2 \right\rangle$.

12. (8 points) Given that $\nabla \times \vec{F} = \langle 0, 0, 0 \rangle$ everywhere on its domain $\mathbb{R}^3 \setminus (\ell_1 \cup \ell_2)$, $\oint_{C_1} \vec{F} \cdot d\vec{s} = 5$, $\oint_{C_2} \vec{F} \cdot d\vec{s} = 8$, and $\oint_{C_4} \vec{F} \cdot d\vec{s} = 3$, find $\oint_{C_3} \vec{F} \cdot d\vec{s}$ and $\oint_{C_5} \vec{F} \cdot d\vec{s}$. Show work to defend your answers.