Test #4A 100 points Math 200 Name:____

Simplify your final answers. Show organized work. Defend all answers.

1) Use a surface integral to find the mass of the sphere $\rho = 1$ if the density is $\delta = x^2 + y^2$ grams / cm^2. (10 points)

2) Use a surface integral to find the flux of $\vec{F} = \langle x, y, y \rangle$ through the graph of $z = 1 - x^2 + y$ if x and y are between -1 and 1. (10 points)

3) Find the flux of $\vec{F} = \langle z, 2, z \rangle$ through the surface parameterized by $\vec{p}(u,v) = \langle uv, u+v, 2v \rangle$ if $u^2 + v^2 \le 1$ and with $d\vec{S}$ with positive $\hat{\mathbf{i}}$ component. (10 points)

4) Evaluate each limit or show why it does not exist. (10 points)

4A) $\lim_{(x,y)\to(0,0)} \frac{x - y^2}{x^2 + y^2}$

4B) $\lim_{(x,y)\to(0,0)} \frac{x^2 + y^2 + 2}{x^2 - y^2 - 1}$

4C) $\lim_{(x,y)\to(0,0)} \left(x^2 + y^2\right)\ln\left(x^2 + y^2\right)$

5) Find and classify all critical points for $f(x, y) = x^3 - 12xy + 8y^3$. (15 points)

6) Use Lagrange Multipliers to find and classify the critical points for $f(x,y)=2y-x$ constrained by $y^2-x^2=1$. Include a sketch of the appropriate level curves. (15 points)

7) Prove $\iint_{Bd(E)} \nabla\times\vec{F}\cdot d\vec{S}=0$ using the Divergence Theorem. (10 points)

8) Use the Divergence Theorem to find the flux of $\vec{F} = \left\langle x^2 y, 3y^2, -2xz^2 \right\rangle$ out of the unit cube with x, y, and z between 0 and 1. (10 points)

9) Use Stokes' Theorem once to find $\iint_S \nabla \times \vec{F} \cdot d\vec{S}$ if S is the intersection of z = 1 and $\rho \leq 2$ oriented up and $\vec{F} = \left\langle y, 2x - z, x \right\rangle$. (10 points)