Test #4B 100 points Math 200 Name:______________________________________

Simplify your final answers. Show organized work. Defend all answers.

1) Find and classify all critical points for $f(x,y) = xy + x^{-1} + 8y^{-1}$. (15 points)

2) Use Lagrange Multipliers to find and classify the critical points for $f(x,y) = yx^2$ constrained by $x^2 + y^2 = 3$. Include a sketch of the appropriate level curves. (15 points)

3) Use Stokes' Theorem **once** to find I = $\int_{Bd(S)} x^2 y\, dx + \frac{x^3}{3}\, dy + xy\, dz$ if S is the intersection of $z = y^2 - x^2$ and $x^2 + y^2 \leq 1$, and S is oriented up. (15 points)

4) Use the Divergence Theorem **once** to find I= $\oiint_{Bd(T)} \vec{F} \cdot d\vec{S}$ if $\vec{F} = \langle 2xy, 3y^2, -2zy \rangle$ and T is the solid in the first octant bounded by the coordinate planes and the plane x + 2y + z = 2. (15 points)

5) Use a surface integral to find the mass of the cylinder $x^2 + y^2 = 4$ if $0 \le z \le 3$ and the density is $\delta = x^2$ grams / cm^2. (10 points)

6) Use cylindrical or spherical coordinates to evaluate I= $\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_0^{4-x^2-y^2} 5x^2 \, dz\, dy\, dx$. (10 points)

7) Use Stokes' Theorem **once** to find I= $\iint_S (\nabla \times \vec{F}) \cdot d\vec{S}$ if $\vec{F} = \langle e^z - y, e^z + x, \cos(xz) \rangle$ and S is the upper hemisphere, $\rho = 1$ with $z \geq 0$ oriented **down**. (10 points)

8) Evaluate each limit or show why it does not exist. Defend your answers. (10 points)

8A) $\lim\limits_{(x,y)\to(\infty,\infty)} \dfrac{e^{\sqrt{x^2+y^2}}}{\sqrt{x^2+y^2}}$

8B) $\lim\limits_{(x,y)\to(0,0)} \dfrac{2x^2+y^2}{x^2+y^2}$

8C) $\lim\limits_{(x,y)\to(0,0)} \dfrac{5-3x-4y}{x^2+y^2+3}$