

Simplify your final answers. Show organized work. Defend all answers.

1) Use a surface integral to find the flux of $\vec{F} = \langle y, 2xy, z - y \rangle$ through the graph of $z = 1 - x^2 + y^2$, oriented up, if x and y are between -1 and 1 . (10 points)

2) Find the mass of the sphere $\rho = 1$ if the density is $\delta = x^2 + y^2$ grams / cm^2 . (10 points)

3) Find the flux of $\vec{F} = \langle x, y, y \rangle$ through the surface parameterized by $\vec{p}(u, v) = \langle uv, u, 2v \rangle$ if $u^2 + v^2 \leq 1$. Orient the surface towards the positive $\hat{\mathbf{i}}$ component. (10 points)

4) Evaluate each limit or show why it does not exist. (10 points)

$$4A) \lim_{(x,y) \rightarrow (0,0)} \frac{x - y^2}{x^2 + y^2}$$

$$4B) \lim_{(x,y) \rightarrow (0,0)} (x^2 + y^2) \ln(x^2 + y^2)$$

$$4C) \lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + y^2 + 2}{x^2 - y^2 - 1}$$

5) Find and classify all critical points for $f(x, y) = 12xy - x^3 - 8y^3$. (15 points)

6) Use Lagrange Multipliers to find and classify the critical points for $f(x, y) = y + x^2$ constrained by $y^2 - x^2 = 1$. Include a sketch of the appropriate level curves. (15 points)

7) Prove $\iint_{Bd(E)} \nabla \times \vec{F} \cdot d\vec{S} = 0$ using the Divergence Theorem. E is a solid. (10 points)

8) Use the Divergence Theorem to find the flux of $\vec{F} = \langle x - y, y^2 + z^2, y - x^2 \rangle$ out of the boundary of the solid cylinder $r \leq 1$ for $0 \leq z \leq 1$. (10 points)

9) Use Stokes' Theorem **once** to find $\iint_S \nabla \times \vec{F} \cdot d\vec{S}$ if S is the intersection of $z \leq 4 - r^2$ and $z = 1$, oriented up, and if $\vec{F} = \langle y, 3x, 2z \rangle$. (10 points)