Test #4 100 points Math 200 Name:____

Simplify your final answers. Show organized work. Defend all answers.

1) Evaluate $\iint_R y^2 \, dA$ using a substitution if R is bounded by y = x, y = 3x, xy = 1, and xy = 3. (15 points)

2) Use Stokes' Theorem **once** to find I= $\iint_S \left(\nabla \times \vec{F}\right) \cdot d\vec{S}$ if $\vec{F} = \left\langle z, x, x^2 + y^2 \right\rangle$ and S is the lower hemisphere $x^2 + y^2 + z^2 = 4$, $z \le 0$, oriented down. (10 points)

3) Verify the Divergence Theorem if $\vec{F} = \langle 2y, 4y, 6z \rangle$ and T is the solid cylinder $x^2 + y^2 \le 1,\ 0 \le z \le 2$.

3A) Evaluate the appropriate triple integral. (5 points)

3B) Evaluate the appropriate surface integral. (10 points)

4) Use the Divergence Theorem to evaluate $\iint_S 4x^2 + 7y^2 - 8z^2 \, dS$ if S is $x^2 + y^2 + z^2 = 4$. (10 points)

5) Find and classify all of the critical points of $f(x,y)=e^{-x^2-y^2}$ using the second partials test. Defend your choice of the words "local" or "global." (10 points)

6) Use Lagrange Multipliers to classify the extreme values of $f(x,y)=x+y$ constrained by $x^2+4y^2=16$. **Draw a picture of level curves to defend and verify your final answer**. (15 points)

7) Evaluate each limit or show why it does not exist. Defend your answers. (15 points)

7A) $\lim_{(x,y)\to(0,0)} \frac{2x\left(e^{3y}-1\right)}{y\sin(x)}$

7B) $\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x^2-y^2}$

7C) $\lim_{(x,y)\to(0,0)} \frac{\sqrt{x^2+y^2}}{e^{\sqrt{x^2+y^2}}-1}$

8) Find the curl and divergence of $\vec{F} = \left(2xy+z^2\right)\hat{\mathbf{i}} + \left(yz+2x^2\right)\hat{\mathbf{j}} + \left(2xz+y^2\right)\hat{\mathbf{k}}$. Label each answer with appropriate notation. (10 points)