Test #4 100 points Math 200 Name:____

Simplify your final answers. Show organized work. Defend all answers.

1) Find and classify all of the critical points of $f(x,y)=\frac{x^3}{3}-xy+y^2$ using the second partials test. Defend your choice of the words "local" or "global." (15 points)

2) Use Lagrange Multipliers to classify the extreme values of $f(x,y)=y^2-x$ constrained by $x^2+4y^2=36$. Draw a picture to defend and verify your final answer. (15 points)

3) Find the curl and divergence of $\vec{F} = \left\langle e^{xy}, \ln(y) - xz, \sin(xy) \right\rangle$. Label each answer with appropriate notation. (10 points)

4) Use Stokes' Theorem **once** to find I = $\oint_{Bd(S)} y\,dx - z\,dy + x\,dz$ if S is the piece of the plane $x + 2y + 5z = 10$, oriented up, that lies in the first octant. (10 points)

5) Use the Divergence Theorem **once** to find I= $\oiint_{Bd(T)} \vec{F} \cdot d\vec{S}$ if $\vec{F} = \left\langle 2xy, z^2, -2y \right\rangle$ and T is the part of the solid **half**-cylinder $r \le 1$, $y \ge 0$, that lies between z = 0 and $z = x^2 + y^2$. (10 points)

6) Use Stokes' Theorem **once** to find I= $\iint_S \left(\nabla \times \vec{F}\right) \cdot d\vec{S}$ if $\vec{F} = \langle z, x^2 + y^2, y \rangle$ and S is $z = 4 - x^2 - y^2,\ z \geq 0$, oriented up. (10 points)

7) Use a surface integral to find the mass of the part of the sphere $x^2 + y^2 + z^2 = 4$ where $0 \leq z \leq 1$ if the density is $\delta = z$ grams / cm^2. (10 points)

8) Evaluate each limit or show why it does not exist. Defend your answers. (10 points)

8A) $\lim_{(x,y)\to(2,1)} \frac{x^2+2x-8y^2}{xy}$

8B) $\lim_{(x,y)\to(0,0)} \frac{e^{\sqrt{x^2+y^2}}-1}{\sqrt{x^2+y^2}}$

8C) $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$

9) Find the volume of the solid that lies above the cone $z=\sqrt{x^2+y^2}$ and below $z=6-x^2-y^2$. (10 points)