Final   100 points   Math 210   Name: ____________________________

Always show work or defend your answers.

1. Write "True" or "False" with reason or counterexample.

   (a) (6 points) If the rank of A is 2 and the rank of B is 1, then the rank of $A+B$ cannot equal 2.

   (b) (6 points) Let P_3 be the vector space of polynomials of degree less than 4, and F be the vector space of functions in terms of k. Then $T : P_3 \to F$ so that $T(f(x)) = \int_0^{10} f(x)e^{-kx}\,dx$ is a linear transformation.

   (c) (6 points) Let $A = LU$ be the LU-decomposition of the n x n matrix A. Then the eigenvalues of A equal the eigenvalues of U.

2. $A = \begin{bmatrix} 1 & 1 & 4 \\ 0 & 1 & 5 \\ 0 & 1 & 5 \end{bmatrix}$ for all parts of this question.

(a) (6 points) Find the eigenvalues and eigenvectors for A. Show work or reasoning.

(b) (4 points) Write the general solution to the system of differential equations $\dfrac{d\vec{u}}{dt} = -A\vec{u}$. Notice the minus sign.

(c) (6 points) Find and simplify $\lim_{n\to\infty} \left(\frac{1}{6}A\right)^n$. Show work.

3. (6 points) $L(x, y) = (x + 4y, 2x + 9y)$ is a linear transformation and $B = (1, 1), (1, 2)$ is a basis for $\mathbb{R}^2$. Find $[L]_B$.

4. (6 points) If $b = 0, 10, 0, 0$ when $t = -1, 0, 1, 2$ respectively, find the best fitting line $\hat{b} = \hat{C} + \hat{D}t$.

5. (6 points) Suppose $\vec{u}$ and $\vec{w}$ are orthonormal and $\vec{b} = 3\vec{u} + \vec{w}$. What is the projection of $\vec{b}$ onto $\vec{u} + \vec{w}$? Answer in terms of $\vec{u}$ and $\vec{w}$ and defend your answer.

6. (6 points) $H = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$ has $H^2 = 4I$. For what values of k is $H + kI$ a positive definite matrix? Defend your answer.

7. (6 points) Find the general solution to $A\vec{x} = \vec{b}$ if $A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 4 & 1 & 4 \\ 3 & 6 & 3 & 9 \end{bmatrix}$ and $\vec{b}$ equals the sum of the columns of A.

8. (6 points) Find the matrix of a linear transformation $L : \mathbb{R}^3 \to \mathbb{R}^3$ that rotates 90° counterclockwise about the x-axis and then reflects across the yz-plane. Show work.

9. $A = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$ for all the problems on this page.

(a) (4 points) Sketch a diagram of the circuit represented by incidence matrix A.

(b) (2 points) What is the dimension of $N(A^T)$? __________

(c) (2 points) What is a basis for $C(A)$? ____

(d) (2 points) $C(A^T) =$ ____

(e) (6 points) Use the Gram-Schmidt process to find an orthonormal basis for the span of the first two columns of A.

10. (6 points) A is a 3x3 symmetric matrix with real entries so that $A\begin{bmatrix}1\\1\\0\end{bmatrix} = \begin{bmatrix}2\\2\\0\end{bmatrix}$, $A\begin{bmatrix}1\\-1\\1\end{bmatrix} = \begin{bmatrix}1\\-1\\1\end{bmatrix}$, and -1 is an eigenvalue. What is the rank-1 form of A? Defend your answer.

11. (8 points) Let $B = \begin{bmatrix}1 & 1\\2 & 2\\2 & 2\end{bmatrix}$. Solve $B\vec{x} = \begin{bmatrix}1\\1\\1\end{bmatrix}$ for $\vec{x}^+$.