Final 200 points Math 210 Name: ____________________

Always show work to defend your answer in a logical and organized fashion unless told otherwise.

1. Write "true" or "false" with reason or counterexample.

   (a) (12 points) Let V be the set of all vectors (x, y) with $xy > 0$. Then V is a vector subspace of $\mathbb{R}^2$.

   (b) (12 points) $L : \mathbb{R}^2 \to \mathbb{R}^2$ so that $L(x, y) = (x^2 - y, y^2)$ is a linear transformation.

   (c) (12 points) If A is a 4 x 4 matrix with eigenvalue λ, then an eigenvalue of $A + 4I$ is $\lambda + 4$.

2. $K = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \\ 1 & 2 & 2 & 1 \end{bmatrix}$ for both parts of this question.

(a) (8 points) Is K positive definite? Defend your answer.

(b) (8 points) Use Cramer's rule to find x_3 if $K\vec{x} = (0, 0, 1, 1)$ and $\vec{x} = (x_1, x_2, x_3, x_4)$.

3. (20 points) Find the projection of the vector $(0, 1, 1, 1)$ onto the space spanned by the vectors $(1, 1, 1, 1)$ and $(-1, 1, 0, 2)$. Use matrices in your work.

4. A is a 2 x 2 matrix with real entries, eigenvalues 0.2 and 1, $N(A - 0.2I) = c(1, 4)$, and $N(A - I) = c(2, 9)$ for all parts of this problem.

(a) (8 points) Is A symmetric? Use a theorem to help defend your answer.

(b) (8 points) Write the solution to the system of differential equations $\dfrac{d\,\vec{u}(t)}{dt} = A\vec{u}(t)$ if $\vec{u}(0) = (1, 5)$.

(c) (8 points) What is $\lim\limits_{n\to\infty} A^n$?

5. (8 points) If $M = \begin{bmatrix} 0.2 & 0.5 \\ 0.8 & 0.5 \end{bmatrix}$ is a Markov matrix modeling a system with 1300 objects, what is the steady state vector?

6. (12 points) $L : \mathbb{R}^2 \to \mathbb{R}^2$ so that $L(x, y) = (2x + 3y, 3x + 2y)$ is a linear transformation, and $B = (2, 1), (1, 1)$ is a basis for $\mathbb{R}^2$. Find $[L]_B$.

7. (20 points) Find the general solution to $A\vec{x} = \vec{b}$ if $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 4 & 2 \end{bmatrix}$ and $\vec{b} = (-7, 2, 4)$. Use matrices in your work whenever possible.

8. (16 points) Find the QR-factorization of $B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}$.

9. (20 points) Find the $X\Lambda X^{-1}$ factorization of $K = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$.

10. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 5 \end{bmatrix}$ for this problem. Use only vectors or matrices made from the rows or columns of the given matrices for your answers below.

(a) (4 points) What is $C(A)$? ______________________________

(b) (4 points) What is a basis for $N(A)^{\perp}$? ______________________________
Recall $N(A)^{\perp}$ is the orthogonal complement of $N(A)$.

(c) (4 points) What matrix augmented with the zero vector represents a system with solution equal to $N(A)$?

(d) (4 points) What matrix augmented with the zero vector represents a system with solution equal to $N(A^T)$?

11. (12 points) Find the rank-1 form for the SVD of $B = \begin{bmatrix} 2 & 2 \\ 2 & 2 \\ 1 & 1 \end{bmatrix}$. Show work.