1. If $\|\vec{u}\| = 5$ and $\|\vec{v}\| = 3$, what are the smallest and largest values of $\|\vec{u} - \vec{v}\|$? What are the smallest and largest values of $\vec{u} \cdot \vec{v}$? Defend your answers.

Write each system as an augmented matrix and then use Gaussian elimination to find the pivots and solutions for the following systems.

2. $\begin{cases} 2x - 4y = 6 \\ -x + 5y = 0 \end{cases}$

3. $\begin{cases} 2x - 4y = -6 \\ -x + 5y = 0 \end{cases}$

4. $\begin{cases} 2x + y = 0 \\ x + 2y + z = 0 \\ y + 2z + t = 0 \\ z + 2t = 5 \end{cases}$

Describe the span of the following vectors as a point, line, plane, or all of $\mathbb{R}^3$. Defend each answer using the words "linearly dependent" or "linearly independent" in your sentence.

5. $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$.

6. $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$.

7. $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

True or False? Defend each answer with a proof or counterexample.

8. If (col 1 B) = (col 3 B), then (col 1 AB) = (col 3 AB).
9. If (row 1 B) = (row 3 B), then (row 1 AB) = (row 3 AB).
10. If (row 1 A) = (row 3 A), then (row 1 ABC) = (row 3 ABC).
11. If B is a nonzero square matrix and AB = B, then A = I.
12. A,B and C are matrices. If AC and BC are defined, then (A+B)C = AC + BC.

13. Use matrices to find a nontrivial linear combination of

$$\vec{u} = (1, 2, 3),\ \vec{v} = (4, 5, 6),\ \text{and}\ \vec{w} = (7, 8, 9)$$

that equals the zero vector. "Nontrivial" means at least one scalar must be nonzero.

Are these vectors independent or dependent?

The three vectors lie in a __________ .

$$S = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}, \vec{s}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \vec{s}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \text{ and } \vec{s}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

14. Find $\vec{b} = 2\vec{s}_1 + 3\vec{s}_2 + 4\vec{s}_3$, and then write $\vec{b}$ as a matrix - vector multiplication.

15. Solve $S\vec{y} = (1, 3, 6)$. What linear combination of the $\vec{s}_i$ is used?

Let A be a 3 x 3 matrix. Write down the 3 x 3 matrix B so that BA

16. subtracts 5 times row 1 of A from row 2 of A but leaves row 1 and row 3 of A the same.

17. subtracts -7 times row 2 of A from row 3 of A but leaves row 1 and row 2 of A the same.

18. multiplies row 3 of A by 5 but leaves row 1 and row 2 of A the same.

19. moves row 1 of A to row 3 of BA, row 3 of A to row 2 of BA, and row 2 of A to row 1 of BA.

Let A be a 3 x 3 matrix. Write down the 3 x 3 matrix B so that AB

20. subtracts 5 times column 1 of A from column 2 of A but leaves column 1 and column 3 of A the same.

21. replaces each column of A with the sum of the other two columns.

22. Write the corresponding system of equations for the matrix product $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix}$. Then use elimination to show the system has no solution. Change the six to a number that makes the system solvable and find **one** solution - there are many possible correct solutions.

23. CAS problem (3 points): use a CAS device for the following. Submit a printed copy of your commands and the answers. Find the rank 1 decomposition of $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 9 & 0 \\ 0 & 4 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$. Find A in two different ways: by multiplying the matrices given above and by multiplying the rank-1 decomposition matrices.

Brief answers

1. $2 \leq \|\vec{u} - \vec{v}\| \leq 8$ and $-15 \leq \vec{u} \cdot \vec{v} \leq 15$.
2. Pivots: 2, 3; Solution: (5,1).
3. Pivots: 2, 3; Solution: $(-5, -1)$.
4. Pivots: 2, 3/2, 4/3, 5/4; Solution: $(-1, 2, -3, 4)$.
5. line
6. plane
7. plane
8. True - give a proof
9. False - give a counterexample
10. True - give a proof
11. False - give a counterexample
12. True - give a proof
13. e.g.: $\vec{u} - 2\vec{v} + \vec{w} = \vec{0}$. Can you find a different one? Dependent; Plane.
14. $\vec{b} = (2, 5, 9)$ and $\vec{b} = S \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$
15. $\vec{y} = (1, 2, 3)$; $(1, 3, 6) = \vec{s}_1 + 2\vec{s}_2 + 3\vec{s}_3$.

In the following answers, $B = I$ except in the specified entry:

16. $B_{21} = -5$
17. $B_{32} = 7$
18. $B_{33} = 5$
19. $B = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$
20. $B = \begin{bmatrix} 1 & -5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
21. $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$
22. Elimination shows $0 = 3$. Change six to three and one solution is $(1, 0, 0)$. Can you find another?