Find the LDU factorization of the following matrices

1. $\begin{bmatrix} 2 & 3 \\ 4 & 11 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 4 & 1 \\ 4 & 12 & 4 \\ 0 & 4 & 0 \end{bmatrix}$

Use LU factorization and back substitution to solve the following systems. Include c in your final answer.

3. $\begin{cases} x + y + z = 5 \\ x + 2y + 3z = 7 \\ x + 3y + 6z = 11 \end{cases}$

4. $A\vec{x} = \vec{b}$ if $A = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 0 & 1 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 2 \\ 11 \end{bmatrix}$

5. Factor $C = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ into LDL^T form.

Find the inverse of each matrix below. Check your answers by multiplying to get the identity matrix.

6. $K = \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}$

7. $D = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$

8. $E = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$

9. Suppose A = LU is 4 x 5 with four pivots such that col 1 + col 3 + col 5 = 0. What is N(A)?

Solve each system and express the solution as a linear combination of independent vectors. Determine which variables are free and if the solution set is a point, line, plane, 3-space, 4-space, or $\mathbb{R}^5$.

10. $\begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 1 & 2 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \vec{0}$

11. $\begin{bmatrix} 2 & 4 & 2 \\ 0 & 4 & 4 \\ 0 & 8 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \vec{0}$

What conditions on constants b_1, b_2, and b_3 make the following systems solvable?

12. $\begin{bmatrix} 1 & 4 & 2 \\ 2 & 8 & 4 \\ -1 & -4 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

13. $\begin{bmatrix} 1 & 4 \\ 2 & 9 \\ -1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

Which of the following subsets of $\mathbb{R}^3$ are vector subspaces? Defend each answer.

14. All vectors (x, y, z) with $x = y$.

15. All vectors (x, y, z) that satisfy $x + y + z = 0$.

16. All vectors (x, y, z) with $x \leq y \leq z$.

17. Do the 2π-periodic real valued functions form a vector space?

18. Do the nonnegative functions form a vector space?

Answer "True" or "False" and defend your answer with a reason or counterexample.

19. A 4 x 4 matrix with a row of zeroes is not invertible.

20. Every n x n matrix with ones on the diagonal is invertible.

21. If the matrix A is invertible then A^{-1} and A^2 are also invertible.

22. A and B symmetric implies AB is symmetric.

23. CAS Problem (3 points): Use a CAS for the following. Turn in a printout of the commands and answers.

    Find the average size for each of the three pivots from 50 random LU factorizations.

    MatLab users can use the [L,U]=lu(rand(3)) command. The pivots' averages should be in descending order since a computer always chooses the largest possible entry for a pivot. Also note that "average size" means you need to find the absolute values of the pivots before averaging.

Brief answers

1. $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 1 & 3/2 \\ 0 & 1 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 4 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

3. $\vec{x} = \begin{bmatrix} 5 \\ -2 \\ 2 \end{bmatrix}$ and $\vec{c} = \begin{bmatrix} 5 \\ 2 \\ 2 \end{bmatrix}$

4. $\vec{x} = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$ and $\vec{c} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

5. $\begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ 0 & -2/3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3/2 & 0 \\ 0 & 0 & 4/3 \end{bmatrix} \begin{bmatrix} 1 & -1/2 & 0 \\ 0 & 1 & -2/3 \\ 0 & 0 & 1 \end{bmatrix}$

6. $K^{-1} = \begin{bmatrix} 7 & -4 \\ -5 & 3 \end{bmatrix}$

7. $D^{-1} = \frac{1}{2} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & -1 \end{bmatrix}$

8. $E^{-1} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$

9. $N(A) = c(1, 0, 1, 0, 1)$.

10. $\vec{x} = c_1 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} 0 \\ 0 \\ -3 \\ 0 \\ 1 \end{bmatrix}$; this is a 3-space with free variables x_2, x_4, and x_5 .

11. $\vec{x} = c \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$; this is a line with free variable x_3.

12. $2b_1 = b_2$ and $b_1 = -b_3$.

13. $b_1 = -b_3$.

14. Yes

15. Yes

16. No

17. Yes

18. No

19. True

20. False

21. True

22. False