25 points | Math 210 | Homework 4 | Sections 4.1, 4.2, 10.1 | 23 problems

1. $S =$ span of (1, 2, 2, 3) and (1, 3, 3, 2). Find a basis for $S^\perp$.

When possible, construct a matrix with the required properties; otherwise, explain why you cannot.

2. $C(A)$ contains $(1, 2, -3)$ and $(2, -3, 5)$, and $N(A)$ contains (1, 1, 1).

3. $C(B^T)$ contains $(1, 2, -3)$ and $(2, -3, 5)$, and $N(B)$ contains (1, 1, 1).

4. $M\vec{x} = \begin{bmatrix}1\\1\\1\end{bmatrix}$ has a solution, and $M^T\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$.

5. $P\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$ and $\begin{bmatrix}1 & 1 & 1\end{bmatrix}P = \begin{bmatrix}1 & 1 & 1\end{bmatrix}$.

6. Draw the FTLA picture for $A = \begin{bmatrix}1 & 2\\3 & 6\end{bmatrix}$ and $B = \begin{bmatrix}1 & 0\\3 & 0\end{bmatrix}$.

Project $\vec{b}$ onto the line through $\vec{a}$, check that the error vector $\vec{e}$ is perpendicular to $\vec{a}$, find the projection matrix $P = \dfrac{\vec{a}\vec{a}^T}{\vec{a}^T\vec{a}}$, and then verify that $P^2 = P$ and $P\vec{b}$ equals the projection.

7. $\vec{b} = \begin{bmatrix}1\\2\\2\end{bmatrix}$, $\vec{a} = \begin{bmatrix}1\\1\\1\end{bmatrix}$.

8. $\vec{b} = \begin{bmatrix}1\\3\\1\end{bmatrix}$, $\vec{a} = \begin{bmatrix}-1\\-3\\-1\end{bmatrix}$.

Project $\vec{b}$ onto $C(A)$ by solving $A^TA\hat{x} = A^T\vec{b}$ and using $\vec{p} = A\hat{x}$. Verify that $\vec{e} = \vec{b} - \vec{p}$ is orthogonal to $C(A)$.

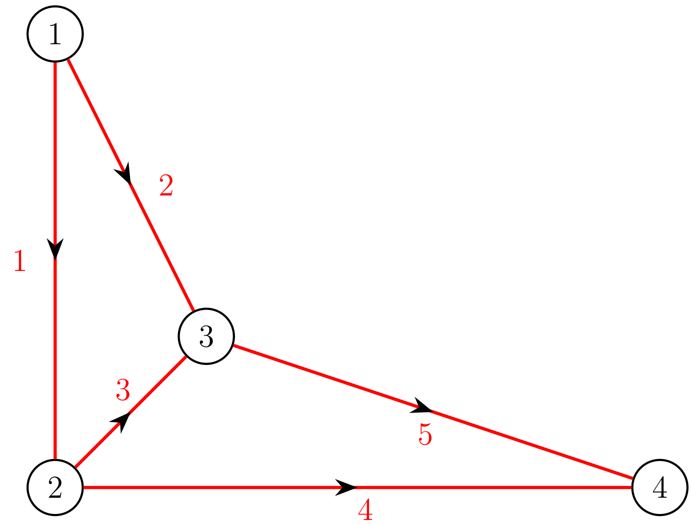
9. $A = \begin{bmatrix}1 & 1\\0 & 1\\0 & 0\end{bmatrix}$, $\vec{b} = \begin{bmatrix}2\\3\\4\end{bmatrix}$.

10. $A = \begin{bmatrix}1 & 1\\1 & 1\\0 & 1\end{bmatrix}$, $\vec{b} = \begin{bmatrix}4\\4\\6\end{bmatrix}$.

11. $A = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\\0 & 0 & 0\end{bmatrix}$. Project $\vec{b} = (1, 2, 3, 4)$ onto $C(A)$. What is the projection matrix P?

12. What linear combination of $(1, 2, -1)$ and $(1, 0, 1)$ is closest to (a) $\vec{b} = (2, 1, 1)$? (b) $\vec{b} = (2, 1, 2)$? What is the error vector $\vec{e}$ in each case?

13. Write down the incidence matrix A for the graph of the circuit shown below. Notice that the node and edge numbers are given. The conductance for edge i is $c_i = |i-3|+1$ and the external current vector is $\vec{f} = (1, -3, 13, -11)$. Use matrices to find the voltages at the nodes and the currents on the edges. Verify that KCL and KVL are adhered to by your answers.



For problems 14 to 17, A is the incidence matrix for a circuit and $A^TA = \begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 2 & 0 & -1 \\ -1 & 0 & 2 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix}$.

14. Draw the circuit with nodes and edges, but without edge numbers or edge orientations.

15. Use your picture of the circuit to find the dimension of the left null space of A and explain how you did it.

16. Let $C = I$, ground node 4, and then solve $A^TA\vec{x} = \begin{bmatrix} 2 \\ 0 \\ 4 \\ -6 \end{bmatrix}$ to find the voltages at the other nodes.

17. Add an orientation and edge number to each edge in your picture of the circuit and find the currents on each edge when the external current vector is $\vec{f} = (2, 0, 4, -6)$ as it was for the last problem.

18. Suppose A is an incidence matrix for a circuit and a basis for $N(A^T)$ is $(0, -1, 1, 0, 0, 1)$,$(1, 0, -1, 1, 1, 0)$. Draw a possible circuit that corresponds with A. Can $A\vec{x} = (1, 1, 1, 0, 0, 0)$ be solved? Defend your answer.

Answer "True" or "False" and then defend your answer.

19. $(1, 1, 1) \cdot (1, 1 - 2) = 0$, so the planes $x + y + z = 0$ and $x + y - 2z = 0$ are orthogonal vector subspaces.

20. The span of $(1, 1, 0, 0, 0)$, and $(0, 0, 0, 1, 1)$ is the orthogonal complement of the span of $(1, -1, 0, 0, 0)$ and $(2, -2, 3, 4, -4)$.

21. If A is the incidence matrix for a connected circuit then $A^T\vec{y} = (1, 4, -5, 7)$has a solution.

22. If $A^T = A$, $A\vec{x} = \vec{0}$, and $A\vec{z} = 5\vec{z}$, then $\vec{x}$ and $\vec{z}$ are perpendicular vectors.

23. CAS Problem (3 points): Use a CAS for the following. Submit a printed copy of the commands and answers. Find the node voltages and edge currents for the circuit with 9 x 6 incidence matrix

$$A = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

diagonal conductance matrix

$$C = \text{diag}(2 \quad 4 \quad 6 \quad 8 \quad 10 \quad 7 \quad 5 \quad 3 \quad 1)$$

and external current vector

$$\vec{f} = (3, 1, 2, -2, -6, 2)$$

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Brief answers

1. $(-5, 1, 0, 1)$, $(0, -1, 1, 0)$
2. possible
3. not possible
4. not possible
5. not possible
6. Each of the fundamental spaces in your picture should be a line in $\mathbb{R}^2$.
7. $\vec{p} = \frac{5}{3}(1, 1, 1)$, $\vec{e} = \frac{1}{3}(-2, 1, 1)$, and $P = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$
8. $\vec{p} = (1, 3, 1)$, $\vec{e} = \vec{0}$, and $P = \frac{1}{11}\begin{bmatrix} 1 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{bmatrix}$
9. $\vec{p} = (2, 3, 0)$, $\vec{e} = (0, 0, 4)$
10. $\vec{p} = (4, 4, 6)$, $\vec{e} = \vec{0}$
11. $\vec{p} = (1, 2, 3, 0)$, $P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

12. (a) $\frac{1}{2}(1,2,-1)+\frac{3}{2}(1,0,1)$, $\vec{e}=\vec{0}$ (b) $\frac{1}{3}(1,2,-1)+2(1,0,1)$, $\vec{e}=\frac{1}{3}(-1,1,1)$.

13. voltages $=\vec{x}=(2,1,3,0)$. currents $=\vec{y}=(3,-2,-2,2,9)$. Note: $\vec{x}$ can be any of $(2,1,3,0)+c(1,1,1,1)$.

14. Change node 1 to 3, 2 to 1, 3 to 4, and 4 to 2 in the picture of the circuit for in problem 13. Ignore the orientations and edge numbers.

15. dim $N(A^T)=2$ because there are two inner loops that form a basis for all loops.

16. $\vec{x}=(2,1,3,0)$.

17. This answer depends on the edge numbers and orientations, but you should get the same components as $\vec{y}=(1,2,-1,1,-3)$ though, perhaps, in a different order and with different signs. Your vector must be orthogonal to any loop vector from $N(A^T)$ since $\vec{y}\in C(A)$.

18. Your circuit should have six edges and two inner loops. $A\vec{x}=(1,1,1,0,0,0)$ can be solved; use the FTLA to explain why.

19. False

20. False

21. False

22. True