25 points Math 210 Homework 9 Sections 8.1-8.3 and Notes 23 problems

If T is a linear transformation from a finite dimensional domain to a finite dimensional range, then $[T]$ denotes the matrix of T with respect to the standard basis, and $[T]_B$ is the matrix of T with respect to basis B.

Explain why the transformation T is not linear or find $[T]$ if it is linear.

1. $T(x, y) = (y, x)$
2. $T(x, y) = (0, x)$
3. $T(x, y) = (0, 1)$
4. $T(x, y) = y - x$
5. $T(x, y) = xy$
6. $T(\vec{v}) = \|\vec{v}\|$
7. $T(x, y, z) = x + y + z$
8. $T(x, y, z) = (x, 2y, 3z)$

9. A linear transformation T has $T(1, 1) = (2, 2)$ and $T(2, 0) = (0, 0)$. Find $T(a, b)$.
10. Let V be the vector space of real valued functions. Is $L : V \to V$ such that $L(f(t)) = f(t)e^t$ a linear transformation? Defend your answer.

Find the range of T (column space of $[T]$) and the kernel of T (null space of $[T]$.)

11. $T(x, y) = (x - y, 0)$
12. $T(x, y, z) = (x, y)$

Let V be the basis $\vec{v}_1, \vec{v}_2, \vec{v}_3$ for $\mathbb{R}^3$ and $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear transformation.

13. Find $[T]_V$ if $T(\vec{v}_1) = \vec{v}_1 + \vec{v}_2 + \vec{v}_3$, $T(\vec{v}_2) = \vec{v}_2 + \vec{v}_3$, and $T(\vec{v}_3) = \vec{v}_3$.
14. What input vector $\vec{v}$ gives $T(\vec{v}) = \vec{v}_1$?
15. What are $T^{-1}(\vec{v}_1)$, $T^{-1}(\vec{v}_2)$ and $T^{-1}(\vec{v}_3)$?
16. What is $\left[T^{-1}\right]$?

17. Draw a picture of T(unit square) if $[T]$ equals the given matrix.
    (a) $D = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $J = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

18. Let $\vec{v}_1 = (-1, 1), \vec{v}_2 = (3, 2)$ and $\vec{w}_1 = (2, 3), \vec{w}_2 = (1, -1)$ be bases V and W respectively for $\mathbb{R}^2$.
    (a) What matrix M changes from V-coordinates to W-coordinates?
    (b) What are the W-coordinates of the point $(15, 10)_V$?

19. Let $\vec{v}_1 = (2,5), \vec{v}_2 = (3,7)$ and $\vec{w}_1 = (1,5), \vec{w}_2 = (1,4)$ be bases V and W respectively for $\mathbb{R}^2$.
(a) What matrix M changes coordinates from basis W to the basis V?
(b) What are the V-coordinates of $(1,2)_W$? Check your answer using the definition of coordinates.
(c) If T is a linear transformation with $T(\vec{v}_1) = \vec{v}_1 + \vec{v}_2$, and $T(\vec{v}_2) = \vec{v}_2 - \vec{v}_1$, then what is $[T]_W$?

Answer "True" or "False" and then defend your answer.

20. If T is a linear transformation then so is T^{-1}.

21. If A is a matrix with real entries, then $A + iI$ is invertible.

22. Find the matrix for the linear transformation L that first rotates 90 degrees counterclockwise about the y – axis, then reflects through the plane z = x, and finally rotates 90 degrees clockwise about the y -axis. What is $L(x,y,z)$ equal to?

23. CAS Problem (3 points): Use a CAS for the following. Submit a printed copy of your commands and answers. Make four plots that include the unit circle and the transformation of the unit circle using the following four matrices: $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, and $D = \begin{bmatrix} 4 & -4 \\ 4 & 4 \end{bmatrix}$. Be sure to use axes large enough to see all of the transformation and the circle.

Brief answers

1. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
2. $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$
3. $T(2,0) \neq 2T(1,0)$
4. $\begin{bmatrix} -1 & 1 \end{bmatrix}$
5. $T(1,1) \neq T(1,0) + T(0,1)$
6. $T(1,1) \neq T(1,0) + T(0,1)$
7. $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$
8. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$
9. $(2b, 2b)$
10. Yes
11. Range $= c(1,0)$; ker $= c(1,1)$
12. Range $= c_1(1,0) + c_2(0,1) = \mathbb{R}^2$; ker $= c(0,0,1)$
13. $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$
14. $\vec{v}_1 - \vec{v}_2$
15. $T^{-1}(\vec{v}_3) = \vec{v}_3$, $T^{-1}(\vec{v}_2) = \vec{v}_2 - \vec{v}_3$, $T^{-1}(\vec{v}_1) = \vec{v}_1 - \vec{v}_2$
16. $\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$

17. (a) D image is a 2 x 1 solid rectangle with vertices (0, 0), (2, 0), (2, 1), and (0, 1).
(b) J image is a solid parallelogram with vertices (0, 0), (1, 0), (2, 1), and (1, 1).

18. (a)$M = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$ (b) $(10, -5)_W$

19. (a) $M = \begin{bmatrix} 8 & 5 \\ -5 & -3 \end{bmatrix}$ (b)$(18, -11)_V$ (c)$[T]_W = \begin{bmatrix} -54 & -34 \\ 89 & 56 \end{bmatrix}$

20. True

21. False

22. $\begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$ $L(x, y, z) = (-z, y, -x)$