Motivation

Have you seen some Linear Algebra in earlier classes? Yes!

Solve $\begin{cases} x + 2y - z = 3 & R1 \\ x - 3y + z = 2 & R2 \\ 2x - y - z = 3 & R3 \end{cases}$

Kill x

$R_2 - R_1$

$$\begin{aligned} x - 3y + z &= 2 \quad (R_2) \\ -x - 2y + z &= -3 \quad (-R_1) \\ \hline -5y + 2z &= -1 \end{aligned}$$

$R_3 - 2R_1$

$$\begin{aligned} 2x - y - z &= 3 \\ -2x - 4y + 2z &= -6 \\ \hline -5y + z &= -3 \end{aligned}$$

$$\Longleftrightarrow \begin{cases} x + 2y - z = 3 & R_1 \\ -5y + 2z = -1 & R_2 \\ -5y + z = -3 & R_3 \end{cases}$$

$R_3 - R_2$

$$\begin{aligned} -5y + z &= -3 \\ 5y - 2z &= 1 \\ \hline -z &= -2 \text{ or } z = 2. \end{aligned}$$

$$\Longleftrightarrow \begin{cases} x + 2y - z = 3 \\ -5y + 2z = -1 \\ z = 2 \end{cases}$$

B/sub

$\Rightarrow \boxed{z = 2}$

$\Rightarrow -5y + 2(2) = -1 \Rightarrow -5y = -5 \Rightarrow \boxed{y = 1}$

$\Rightarrow x + 2(1) - (2) = 3 \Rightarrow \boxed{x = 3}$

OR $\boxed{(3, 1, 2)}$

Write $\begin{cases} x+2y-z=3 \\ x-3y+z=2 \\ 2x-y-z=3 \end{cases}$ using matrices and vectors and solve the problem again with an "augmented" matrix.

$$\Longleftrightarrow \underbrace{\begin{bmatrix} 1 & 2 & -1 \\ 1 & -3 & 1 \\ 2 & -1 & -1 \end{bmatrix}}_{\text{matrix}} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{\text{vectors}} = \underbrace{\begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix}}_{\text{(column vectors)}} \Longleftrightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 1 & -3 & 1 & 2 \\ 2 & -1 & -1 & 3 \end{array}\right].$$

Solve the system:

R_2+R_1

R_3+2R_1

$$\begin{array}{c|c} & \begin{array}{cccc} x & y & z & \end{array} \\ \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} & \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 1 & -3 & 1 & 2 \\ 2 & -1 & -1 & 3 \end{array}\right] \end{array} \xLeftrightarrow[R_3-2R_1]{R_2-R_1} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -5 & 2 & -1 \\ 0 & -5 & 1 & -3 \end{array}\right]$$

$$\xLeftrightarrow{R_3-R_2} \begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -5 & 2 & -1 \\ 0 & 0 & -1 & -2 \end{array}\right] \end{array}$$

R_3+R_2

$\Rightarrow x+2(1)-(2)=3 \Rightarrow \boxed{x=3}$

$\Rightarrow -5y+2(2)=-1 \Rightarrow \boxed{y=1}$

$\Rightarrow \boxed{z=2}$

Solve Ax=b where

transpose $\begin{bmatrix}3 & 2 & 3\end{bmatrix}^T = \begin{bmatrix}3\\2\\3\end{bmatrix}$

```
A=[1 2 -1; 1 -3 1; 2 -1 -1] , b = [3 2 3]'
```

OR $b = [3;2;3]$

```
A = 3×3
      1     2    -1
      1    -3     1
      2    -1    -1
b = 3×1
      3
      2
      3
```

```
x=A\b
```

← Solving $A\vec{x} = \vec{b}$

```
x = 3×1
      3
      1
      2
```

If we add a column for another variable to this system, we would have a general solution consisting of an infinite amount of solutions. MatLab uses projection (chapter 4) to give just one solution.

```
c=[1 1 1]' , K=[A c] , y=K\b
```

solving $K\vec{y} = \vec{b}$

```
c = 3×1
      1
      1
      1
```

$$\begin{array}{cccc} x & y & z & w \end{array}$$

```
K = 3×4
      1     2    -1     1
      1    -3     1     1
      2    -1    -1     1
y = 4×1
    3.0000
    1.0000
    2.0000
         0
```

Why are there decimals in this answer, but not the first one? (3, 1, 2, 0) is also a solution of the next system, but the program gives a different answer:

```
P=[A b], m = P\b
```

Solving $P\vec{m} = \vec{b}$

```
P = 3×4
      1     2    -1     3
      1    -3     1     2
      2    -1    -1     3
m = 4×1
         0
   -0.0000
   -0.0000
    1.0000
```

note: $(3,1,2,0)^T$ is also a solution.