Gaussian Elimination (HW #1)

You have solved systems like $\begin{array}{l} 2x + 4y + 6z = 14 \\ x - 3y + z = -5 \\ 4x + y - z = 1 \end{array}$ using row operations that allow you to switch the order of the equations (and variables), multiply each row by a constant, and

replace the ith equation R_i with $R_i - kR_j$ where k is any constant and $i \neq j$.

Gaussian Elimination uses only the last one. It is called the **elementary row operation**. A computer uses the switching of equations and variables to reduce round-off error, and we will assume that job is all done when we are given a system in math 210.

Gaussian Elimination is the process of producing a lower triangle of zeroes in the system. We will use matrices so that we don't have to write the variables and equal signs over and over again.

Use Gaussian Elimination and matrices to solve $\begin{array}{l} 2x + 4y + 6z = 14 \\ x - 3y + z = -5 \\ 4x + y - z = 1 \end{array}$. Keep track of the elementary row operations used.

The **pivots** for this system are 2, -5, and $-51/5$; pivots are the leading nonzero entries in each row after Gaussian Elimination has been performed and

$$\begin{bmatrix} 2 & 4 & 6 & 14 \\ 0 & -5 & -2 & -12 \\ 0 & 0 & -51/5 & -51/5 \end{bmatrix}$$

is the **row echelon form** (ref) of the system.

Questions this course will answer:

1) If ref $= \begin{bmatrix} 2 & 4 & 6 & 14 \\ 0 & -5 & -2 & -12 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, how do we write the solution set and what does it mean for a circuit?

2) If ref $= \begin{bmatrix} 2 & 4 & 6 & 14 \\ 0 & -5 & -2 & -12 \\ 0 & 0 & 0 & 2 \end{bmatrix}$, what is the "best" solution and how might it influence college admissions?

Two observations will move us towards answering these questions:

1) $\begin{aligned} 2x + 4y + 6z &= 14 \\ x - 3y + z &= -5 \\ 4x + y - z &= 1 \end{aligned}$ can be written using a **linear combination of vectors:**

$$x \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} + y \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix} + z \begin{bmatrix} 6 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 14 \\ -5 \\ 1 \end{bmatrix}.$$

2) The matrix of coefficients $A = \begin{bmatrix} 2 & 4 & 6 \\ 1 & -3 & 1 \\ 4 & 1 & -1 \end{bmatrix}$ can be factored!

$$A = LU = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 2 & 7/5 & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 & 6 \\ 0 & -5 & -2 \\ 0 & 0 & -51/5 \end{bmatrix}.$$

L is a **lower triangular matrix** and U is an **upper triangular matrix**. L is formed using the factors from the elementary row operations in Gaussian Elimination. We say the **rank** of A is three, the number of pivots.