Matrices - Vectors Generalized (HW #1)

An m x n matrix is an array of numbers with m rows and n columns. m x n is the **size** of the matrix.

$M = \begin{bmatrix} 2 & 1 & 4 \\ 5 & -6 & 7 \end{bmatrix}$ has size __________.

M_{ij} = the entry of M in the ith row and jth column[1].

$M_{23} =$ __________ $M_{12} =$ __________

$\vec{b} = \begin{bmatrix} 5 \\ 24 \end{bmatrix}$ has size __________. When a matrix is a vector, we use $\vec{b}_i = \vec{b}_{i1}$, so

$\vec{b}_1 =$ __________ and $\vec{b}_2 =$ __________.

Addition, subtraction, and scalar multiplication of matrices is performed component-by-component just as with vectors. Using our new notation:

$(A \pm B)_{ij} = A_{ij} \pm B_{ij}$ if A and B are matrices with the same size, and

$(cA)_{ij} = cA_{ij} = A_{ij}c = (Ac)_{ij}$ if c is a scalar.

Simplify $$2\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} 5$$

[1]Matlab uses $M(i, j)$ and frequently people use m_{ij} too.

Multiplication of Matrices!

We have already defined the dot product of $\begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$ to be $\begin{bmatrix} 1 & 3 & -2 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$. Notice that the number of columns in $\begin{bmatrix} 1 & 3 & -2 \end{bmatrix}$ equals the number of rows in $\begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$. The result is a scalar, which is also a 1 x 1 matrix. To generalize this to any pair of matrices, we repeat multiplication of rows times columns. We say the ijth entry of AB is (Row i A)(Col j B). Matlab uses $A(i,:)$ for row i of A and $B(:,j)$ for column j of B. Using this notation we can write:

If A is m x n and B is n x p, then AB is an m x p matrix with

$$(AB)_{ij} = A(i,:)B(:,j)$$

Let $A = \begin{bmatrix} 2 & 5 & 0 & 0 \\ 6 & 9 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 2 & 4 \\ 1 & 0 & 0 & 3 & 5 \end{bmatrix}$.

If $M = AB$, what is the size of M?

$M_{23} =$

$M_{42} =$

$M(:,3) =$

Only one column of B is needed: Col 3 $AB = A$ (Col 3 B). **In general, Col j $AB = A$ (Col j B), a linear combination of the columns of A.**

$M(2,:) =$

In general, Row i AB = (Row i A) B, a linear combination of the rows of B.

Let I_n equal the n x n matrix with ones on the diagonal and zeroes everywhere else. Let

$M = \begin{bmatrix} 2 & 5 & 7 & 2 & 5 \\ 6 & 9 & 15 & 6 & 9 \\ 2 & 0 & 1 & 4 & 5 \\ 0 & 2 & 1 & 2 & 5 \end{bmatrix}$. What is I_4M and MI_5 and why?

Conventually I_n is denoted generically by I and n is determined by the size of the other matrices so that multiplication is defined. I is called the **identity matrix**. So from now on the previous question will be stated "what is IM and MI?"

Create a 3 x 4 matrix A, let $E = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and let $P = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$. What is EA and how do you know? Any matrix with ones on the diagonal and at most one other nonzero entry is called an **elementary matrix**.

Computers use elementary matrices and multiplication to perform elementary row operations on a matrix.

What is AP and how do you know? Any matrix equal to I with rows permuted is called a **permutation matrix**.

Multiplying a matrix on the right by a permutation matrix permutes the columns of the matrix. What happens if the permutation matrix is on the left? Create a matrix to test this out with P.

Rank - 1 Decompostion

If AB is defined, then A and B can be partitioned into blocks so that the partition of block columns of A is the same as the partition of block rows of B. We then multiply the blocks together to get the final product. Rank-1 decomposition is an important application of block multiplication where each block of the first matrix is a column and that of the second is a row. In the example below, the "block size" of A is 1 x 2 and that of B is 2 x 1.

$$A = \left[\begin{array}{c|c} 1 & 2 \\ 3 & 4 \end{array}\right] \text{ and } B = \left[\begin{array}{ccc} 7 & 8 & 2 \\ \hline 9 & -1 & 3 \end{array}\right].$$

What blocks multiply together? Write the product as the sum of the products of the correponding blocks. This is the rank-1 decomposition of AB. Calculate the blocks and add to verify it is equal to AB calculated the original way.

Find the rank-1 decomposition of $\left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & 0 \\ 0 & 6 \end{bmatrix}\right)\begin{bmatrix} 7 & 8 \\ 9 & -1 \end{bmatrix}$

True (always true) or False (sometimes false)? Give a reason if "True" and a counterexample if "False."

A) $AB = BA$ if both matrices are n x n.

B) $A(B + C) = AB + AC$ if A is m x n and B and C are n x p.

C) $A(BC) = (AB)C$ if A is m x n, B is n x p, and C is p x w.