LU-Factorization (HW #2)

Find the LU-factorization and LDU-factorization of $A = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 2 & 1 \\ 1 & 6 & 7 \end{bmatrix}$.

We can prove this algorithm always works. Use elementary matrices to show in detail why the algorithm works in this particular example.

Use the LU-factorization of A and back substitution to solve $A\vec{x} = \vec{b}$ if $\vec{b} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$. This method is much faster than Gauss-Jordan elimination, especially for larger systems.

A is invertible. How do we know? Find the second column of A^{-1} using LU and back substitution.

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You cannot find the LU-factorization of $B = \begin{bmatrix} 0 & -2 & 1 \\ 1 & 4 & 5 \\ 1 & 6 & 7 \end{bmatrix}$ since 0 is in the first pivot spot. Instead find the LU-factorization of PB where P is a permutation matrix that switches row 1 with row 2. What must P be?

Now solve $B\vec{x} = \begin{bmatrix} 1 \\ 4 \\ 6 \end{bmatrix}$ using LU-factorization and back substitution.