Transposes (HW #2)

The **transpose of** A, denoted A^T, is defined by $A^T_{ij} = A_{ji}$. This means $A^T(j,:) = A(:,j)$ and $A^T(:,i) = A(i,:)$, so rows are transposed to columns and columns are transposed to rows. What is $\left(A^T\right)^T$?

$$\begin{bmatrix} 2 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}^T =$$

$$(2,1,3)\cdot(4,2,-1) = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}^T \begin{bmatrix} 4 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 4 & 2 & -1 \end{bmatrix}^T = \langle (2,1,3),(4,2,-1) \rangle$$ [1]

A is **symmetric** if $A^T = A$. Make a few examples of symmetric matrices. What do you notice about the size of a symmetric matrix?

P is a **permutation** if its rows are the permuted rows of an identity matrix. Make several examples. Calculate PP^T in each case. What do you notice?

[1] Matrices are functions: the input $\vec{x}$ is sent to the output $\vec{b}$ in the equation $A\vec{x} = \vec{b}$. In general, functions do not have finite matrices associated with them, so there is another equivalent definition of transpose that is more general, works for functions, but is harder to understand: for m x n A, $\langle A\vec{x}, \vec{y}\rangle = \langle \vec{x}, A^T\vec{y}\rangle$ for all $\vec{x} \in \mathbb{R}^n$ and all $\vec{y} \in \mathbb{R}^m$. We won't use this definition much in our class.

True (always true) or False (at least once)? Defend your answer.

1) $(AB)^T = B^T A^T$

2) $A^T A$ is a symmetric matrix.

3) $\left(A^T\right)^{-1} = \left(A^{-1}\right)^T$

True (always true) or False (at least once)? Defend your answer.

4) If A and B are symmetric n x n matrices, then AB is symmetric.

5) If P is a permutation then $P^T = P^{-1}$.