The Null Space of a Matrix (HW #2)

Solve $A\vec{x} = \vec{0}$ if $A = \begin{bmatrix} 1 & 2 & 4 \\ -1 & 0 & -2 \\ 0 & 4 & 4 \end{bmatrix}$. Label the pivot columns and free columns.

The solution set is the **null space of A**, denoted $N(A)$.

Your textbook calls $\begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$ a "special solution" because later we will learn it forms a **basis** for the solution set.

The **null space** of a matrix A is the set of all solutions to the equation $A\vec{x} = \vec{0}$ and is denoted $N(A)$.

$N(A)$ is calculated using Gaussian elimination and then generating a set of independent "special solutions" that span the solution set. I prefer to do this by setting one of the free variables to 1 and the others to 0; there is a "special solution" for each free variable.

Find $N(A)$ if $\left[A \,|\, \vec{0}\right]$ becomes $\begin{bmatrix} 0 & 1 & 2 & 0 & | & 0 \\ 0 & 0 & 1 & 1 & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \end{bmatrix}$ using elimination.

$N(A)$ is our first example of a "vector subspace." It is a set of vectors that satisfies two conditions:

If $\vec{x}$ and $\vec{y}$ are in $N(A)$, and c is a real number, then

1) $\vec{x} + \vec{y} \in N(A)$ so that $N(A)$ is "closed under vector addition"(CUVA). Prove it.

2) $c\vec{x} \in N(A)$ so that $N(A)$ is "closed under scalar multiplication"(CUSM). Prove it.