The General Solution (HW #3)

The general (or "complete" or "total") solution for the system $A\vec{x} = \vec{b}$ is the set of all solutions for the system.

Theorem: If $\vec{x}_p$ is any solution to the system $A\vec{x} = \vec{b}$, then the general solution for the system is the set $\vec{x}_p + N(A)$.

Proof: We must show that if $A\vec{x} = \vec{b}$, then there exists $\vec{n} \in N(A)$ so that $\vec{x} = \vec{x}_p + \vec{n}$.

But $A(\vec{x} - \vec{x}_p) = \vec{0}$ so . . . finish the proof.

The tactics for solving $A\vec{x} = \vec{b}$ are then:

1) Use Gaussian Elimination on the augmented system.

2) Find a particular solution by setting the free variables to zero.

3) Find $N(A)$ by replacing the reduced $\vec{b}$ with the zero vector.

4) Writing the final answer using the particular solution and a span of the "special solutions."

Find the general solution for $A\vec{x} = \vec{b}$ if $A = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 2 & 1 & 4 & 0 & 2 \\ 0 & 1 & 2 & 1 & -1 \\ 1 & 0 & 1 & -1 & 2 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 2 \\ 9 \\ 6 \\ 1 \end{bmatrix}$.

Find the general solution for $B\vec{x} = \vec{b}$ if $B = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 2 & 0 & 1 \\ 0 & 1 & -1 & 0 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$.