Change of Basis (HW #9)

Coordinates are not in the textbook.

$B = (1,1), (1,-1)$ is a basis for $\mathbb{R}^2$ - order matters!

$$(2,3)_B = 2(1,1) + 3(1,-1) = (5,-1).$$

$(2,3)_B$ are the B**-coordinates**

and $(5,-1)$ are the **standard coordinates** for the standard basis since $(5,-1) = 5(1,0) - (0,1)$.

The **standard basis** for $\mathbb{R}^n$ are the columns of the identity matrix.

Choose a new basis C for $\mathbb{R}^2$. What are the C-coordinates of $(2,3)_B$? The following notation helps me:

$$\mathbb{R}^2_B \longrightarrow \mathbb{R}^2 \longrightarrow \mathbb{R}^2_C \tag{1}$$

If $f : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation, $f(1,1) = (4,-2)$, and $f(1,-1) = (-3,1)$ what is $[f]$?

What is $[f]_B$? The notation introduced earlier can be modified into a box and is very helpful.

As time allows:

Find $[f]_C$ if C is the basis $(2, -2), (1, 1)$.

We have already unknowingly studied change of basis. $A = X\Lambda X^{-1}$ is a change of basis and $A = U\Sigma V^T$ is a change of basis. We changed basis so that we could find the half length and major axes of a rotated ellipse.

This last example illustrates how many problems have a "best basis." the Fourier series uses the Fourier basis for instance. Jordan Canonical form is another famous consequence of a choice of basis using eigenvectors and, when there are not enough, generalized eigenvectors. And, finally, we used the Gram-schmidt process to create a best basis.