Test 1 100 points Math 210 Name: ______________________________

Always show work to defend your answer in a logical and organized fashion unless told otherwise.

1. $A = \begin{bmatrix} -1 & 3 \\ 2 & -1 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ for all parts of this question.

   (a) (5 points) Without work, write the rank-1 decomposition of AB.

   (b) (5 points) Without work, give a linear combination of columns of A that is equal to column 1 of AB.

   (c) (5 points) Without work, write a product of a row vector and a column vector equal to $(AB)_{23}$.

2. (10 points) Solve a system using augmented matrices and Gaussian Elimination to find a linear combination of $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$ that equals $\begin{bmatrix} 4 \\ 6 \\ 6 \end{bmatrix}$.

3. $A = \begin{bmatrix} -1 & 2 & 1 & 0 & 2 \\ 3 & -6 & 1 & 4 & 2 \\ 1 & -2 & 1 & 2 & 2 \end{bmatrix}$ for all parts of this question.

(a) (15 points) What is the LU–factorization of A?

(b) (5 points) What are the pivots? ______________

(c) (5 points) Use U to calculate $N(A)$.

(d) (5 points) Is $N(A)$ a point, line, plane, 3-space, 4-space, or 5-space? ______________

4. The LU - factorization of B is $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & -3 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ for each part of this question.

(a) (10 points) Use the **back substitution method** to solve $B\vec{x} = (1, 10, -13)$.

(b) (5 points) Without work, write a linear combination of columns of L equal to column 2 of B.

(c) (10 points) Find the first column of B^{-1}.

5. (10 points) Let S be equal to the set of all vectors in $\mathbb{R}^3$ of the form $(x, 0, 2x)$. Is S a vector subspace of $\mathbb{R}^3$? Be sure to defend your answer completely.

6. Are the following statements always true or sometimes false? Defend your answers.

   (a) (5 points) If row 3 of B equals row 2 of C then row 3 of BA equals row 2 of CA.

   (b) (5 points) If $\vec{x}$ is a n x 1 vector and A is an m x n matrix, then $\vec{x}^T A^T A\vec{x} = \|A\vec{x}\|^2$.