Test 2 100 points Math 210 Name: _____

1. (10 points) $M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 2 & 2 & 1 & 0 \\ 2 & 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$. The first and second blank are intentionally prefaced by "Basis for," so the answers should be in a different form than the other two.

Basis for $N(M)$ is _____ Basis for $N(M^T)$ is _____

$C(M) =$ _____ $C(M^T) =$ _____

2. (15 points) In each part, write the general solution for the system $A\vec{x} = \vec{b}$ for the given A and $\vec{b}$.

(a) $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 4 \\ 3 & 0 & 6 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 3 \\ 6 \\ 5 \end{bmatrix}$.

(b) $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 4 \\ 3 & 0 & 6 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$.

3. (5 points) M is a 12 x 24 matrix with 14 free variables. What are the dimensions of the fundamental spaces of M?

Dim $C(M) =$ _____ Dim $N(M) =$_____ Dim $C(M^T) =$ _____ Dim $N(M^T) =$_____

4. (5 points) $V = \{(a, 3b, b, 2a) | a, b \in \mathbb{R}\}$ is a vector subspace. Find a basis for V.

5. In each part of this question, construct a matrix A that satisfies the constraints, or use the FTLA to explain why it is not possible.

   (a) (10 points) The column space of the matrix A has basis $(1, 2, 1), (1, 1, 1)$ and the null space has basis $(1, 0, -1)$.

   (b) (10 points) $\vec{x}^T A = \begin{bmatrix} 3 & 1 & 3 \end{bmatrix}$ has a solution and $A \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

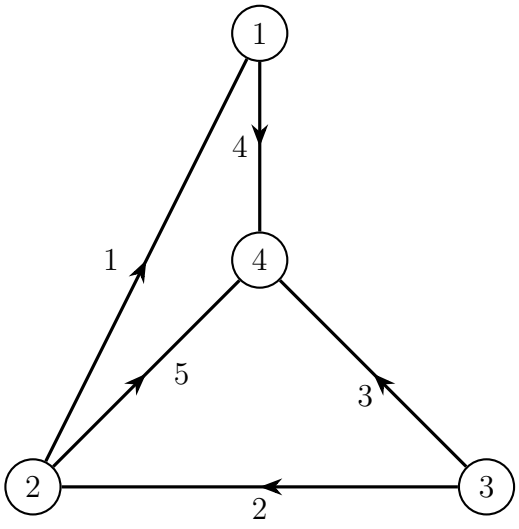
6. (a) (15 points) Set up and solve a system of equations to find the projection of $(1, 0, 0, 1)$ onto the space spanned by $\vec{v} = (1, 1, 0, 0)$ and $\vec{w} = (0, 0, 1, 1)$. Show work.

(b) (10 points) What is the projection matrix P for the vector space spanned by $\vec{v}$ and $\vec{w}$?

7. (5 points) Is the following statement always true or sometimes false? Defend your answer.

B and B^T always have the same rank.

8. Use the circuit shown below for each part. The edge numbers are labeled on the picture of the circuit. A is the incidence matrix.



(a) (5 points) What is A?

(b) (5 points) Find a basis for $N(A^T)$ using the picture of the circuit.

(c) (5 points) If the conductance on each edge i of the above circuit is $i + 2$ mhos, set up, **but do not solve**, an augmented matrix representing a system of equations with solution equal to the voltages on the nodes if the fourth node is grounded and the external current vector is $\vec{f} = (3, -2, 4, -5)$.