

**Always show work that completely defends your answers unless stated otherwise.**

1. (15 points) Factor  $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}$  into QR form.

2. (10 points)  $B$  is a  $3 \times 3$  matrix and  $|B| = 10$ . Let  $B_i = \text{Row } i \text{ B}$ . What is  $|K|$  if  $K = \begin{bmatrix} 2B_1 - 3B_2 \\ 4B_2 - 5B_3 \\ B_1 - B_2 - B_3 \end{bmatrix}$ ?

3. (10 points) Find the best fitting line  $b = C + Dt$  if  $b = 1, 2, 4$  when  $t = 0, 1, 2$  respectively.

4. (15 points) Find all eigenvalues and eigenspaces for  $A = \begin{bmatrix} 2 & 9 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ .

5. (15 points) Use the inner product  $\langle f, g \rangle = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)g(x) dx$  to find the projection of the  $2\pi$ -periodic function defined by  $f(x) = \begin{cases} 1 & \text{if } 0 \leq x \leq \pi \\ 0 & \text{if } -\pi < x < 0 \end{cases}$  onto the span of  $\{1, \sin(x), \cos(x)\}$ .

6. (10 points) The eigenvalues of 3 by 3 matrix  $A$  are  $-1, 0,$  and  $2$ . What are the three eigenvalues of  $B = A^2 + 4A - 2I$ ? Defend your answer by rewriting  $B\vec{x}$  in terms of  $\lambda$  if  $\vec{x}$  is an eigenvector of  $A$  with eigenvalue  $\lambda$ .

7. The first column of a matrix  $K$  is  $\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$  and the cofactor matrix of  $K$  is  $C = \begin{bmatrix} 4 & a & b \\ 2 & 2 & 2 \\ 1 & 3 & 5 \end{bmatrix}$ .

(a) (5 points) What is the determinant of  $K$ ?

(b) (5 points) What are  $a$  and  $b$  equal to?

(c) (5 points) Solve  $K\vec{x} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$  for  $x_2$  if  $\vec{x} = (x_1, x_2, x_3)$ .

8. (10 points) Answer "always true" or "sometimes false" for the following statement. Defend your answer completely.

**If  $A$  is an invertible  $n \times n$  matrix, then  $|A| \neq 0$ .**