Test 4 100 points Math 210 Name: ______________________

Always show work to defend your answer in a logical and organized fashion unless told otherwise.

1. (15 points) Factor A into the $X\Lambda X^{-1}$ form if $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & 0 \\ 2 & 0 & 1 \end{bmatrix}$.

2. (a) (5 points) Let c be a constant real number. What symmetric matrix A is associated with the quadratic polynomial $cx^2 + 2cy^2 + z^2 + 2cxy - 2cyz$?

   (b) (5 points) For what values of c is the matrix A positive definite? Defend your answer.

3. (15 points) Solve the system $\dfrac{dx}{dt} = 3x - 7y$ and $\dfrac{dy}{dt} = x - 5y$ if $x(0) = 1$ and $y(0) = 7$ using matrices.

4. (10 points) 11,000 people are watching a baseball game. Before each pitch, 60% of those looking at their cell phone stop looking at it, and 50% of those not looking at the cell phone start looking at it and continue to until the next pitch. Estimate the number of people looking at their cell phone during the last pitch of the game. There are usually more than 200 pitches each game. Defend your answer using eigenvalues in your work.

5. $A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 1 \end{bmatrix}$ for both parts.

(a) (15 points) Factor A into the rank-1 form of the SVD.

(b) (10 points) Find $\vec{x}^+$ for $A\vec{x} = \vec{b}$ if $\vec{b} = (0, 0, 8)$.

 # 6 - 8 are cumulatively worth 25 points. Your two best answers will be graded out of 10 points each and the last one will be graded out of 5 points.

6. If A is a real symmetric matrix and is similar to B, is B also always symmetric? Defend your answer.

7. If a 2 x 2 symmetric matrix M with real entries has eigenvalues -1 and 1 with $N(M+I) = \text{ span } \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, then what is $N(M-I)$ and what is M? What theorem allows you to find these? Write M as a single matrix in your final answer.

8. Is the following statement always true or sometimes false?
   If the eigenvalues of a 2 x 2 matrix K are 1 and 2 then the singular values of K are also 1 and 2.