

Final 200 points Math 220 Name: _____
Always show work unless told otherwise.

1. (5 points) Write the general solution for $x^2y'' + xy' + (9x^2 - 25)y = 0$. No work is required.
2. (10 points) Write the general solution for $x^2y'' + xy' + 9y = 0$.
3. (20 points) Solve $y'' - 3y' + 2y = 12e^{-t}$ if $y(0) = 5$ and $y'(0) = 3$.

4. (15 points) Find the charge on the capacitor at time t , $q(t)$, of a circuit modeled by the differential equation $q'' + 2q' + 5q = 2t\delta(t - 4)$ if $q(0) = 0$ and $q'(0) = 0$.
5. (15 points) Solve $y'(t) = (t - 2)H(t - 2) + y(t) * \cos(t)$ if $y(0) = 1$.
6. (10 points) Solve $u_t = 2u_{xx}$ if $u(0, t) = 0 = u(\pi, t)$, $u(x, 0) = 4 \sin(3x)$. Show the steps discussed in lecture.

7. (a) (10 points) If the general solution to a wave equation is $y(x, t) = \sum_{n=1}^{\infty} c_n \cos(nt) \sin(nx)$ and $y(x, 0) = x$ for $x \in [0, \pi]$ what is the solution?

- (b) (5 points) What are the two boundary conditions and the other initial condition for this wave equation, and how do you know?

8. (15 points) Use the substitution $u = \frac{y}{x}$ to find an implicit solution for $y' = \frac{y}{x} + \sec\left(\frac{y}{x}\right)$. Assume $x > 0$.

9. $\frac{d\vec{u}}{dt} = A\vec{u}$ is a 2 x 2 system of equations with an eigenvalue of i for A and corresponding eigenvector $\begin{bmatrix} 2 + i \\ 1 - i \end{bmatrix}$.

(a) (10 points) Write the general solution for this system.

(b) (5 points) The phase diagram for this system generates a phase plane with spirals. Do the spirals rotate counterclockwise or clockwise? How do you know this?

10. (15 points) Solve $u_{xx} + u_{yy} = 0$ if $u_x(0, y) = 0 = u_x(\pi, y)$, $u(x, 0) = 0$, and $u(x, 1) = 1 + 2 \cos(4x)$. Show the steps discussed in lecture.

11. (15 points) Find a particular solution for $\frac{d\vec{u}}{dt} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \vec{u} + \begin{bmatrix} 8 \\ 8t \end{bmatrix}$ using variation of parameters. Simplify your answer.

12. (15 points) Find and classify the critical points for the nonlinear system $\begin{cases} x' = x(y - 4) \\ y' = (y + 2)(xy - 1) \end{cases}$

13. (15 points) A tank initially contains 10 liters of water with no salt. A solution containing 3 g of salt per liter is pumped into the tank at the rate of two liters per second, and the mixture - kept uniform by stirring - is pumped out at one liter per second. **Set up a first order differential equation** in terms of the amount of salt at any time t and **solve it**.

14. (20 points) Use the Frobenius method to find the first three nonzero terms of one solution for

$$x^2y'' + x^3y' - 6y = 0.$$