20 points Math 220 Homework 2 Sections 1.5, 1.7, 1.8, 2.1-2.3 18 problems

Most of these problems are from Jiri Lebl's "Notes on Diffy Q's, Differential Equations for Engineers."

1. A thermometer is removed from a room where the temperature is 70° F and is taken outside, where the air temperature is 10°F . After one-half minute the thermometer reads 50°F . What is the reading of the thermometer at t = 1 minute? How long will it take for the thermometer to reach 15°F ?

2. A small metal bar, with initial temperature 20°C , is dropped into a large container of boiling water. How long will it take the bar to reach 90°C if it is known that its temperature increases 2°C during the first second of elapsed time? How long will it take the bar to reach 98°C ?

Solve using a substitution.

3. $\dfrac{dy}{dx} + y = xy^2$ if $y(0) = 1/3$

4. $\dfrac{dy}{dx} - \dfrac{y}{x} = 1 + \dfrac{y^2}{x^2}$

5. $x\dfrac{dy}{dx} + 6y = 3xy^{4/3}$

6. $xy' - y - \dfrac{x^3}{y^2}$ if $y(1) = 1$

7. Show $(x+y)^2dx + (2xy + x^2 - 1)dy = 0$ is exact and then solve it implicitly if $y(1) = 1$.

8. Show $\left[x^2e^{x^2+y}(2x^2+3) + 4x\right] dx + (x^3e^{x^2+y} - 12y^2)dy = 0$ is exact and then find the general implicit solution.

9. Find the general real solution for $y'' + 4y' + 2y = 0$.

10. Find the general real solution for $y'' - 6y' + 9y = 0$.

11. Solve $2y'' + y' + y = 0$ if $y(0) = 1$ and $y'(0) = -2$

12. Solve $2y'' + y' - 3y = 0$ if $y(0) = a$ and $y'(0) = b$

13. Solve $z''(t) = -2z'(t) - 2z(t)$ if $z(0) = 2$ and $z'(0) = -2$

14. Find the general real solution for $y^{(5)} - y^{(4)} = 0$

15. Suppose that the characteristic equation of a third order differential equation has roots 3 and $\pm 2i$

    a) What is the characteristic equation?

    b) Find the corresponding differential equation.

    c) Find the general solution.

16. e^x and e^{-x} form a basis for the real general solution set for what differential equation? Prove that $\cosh(x)$ and $\sinh(x)$ form a basis for the same differential equation.

17. Is the set of functions $\cos(x)$, $\sin(x)$, e^x linearly independent? Justify.

18. CAS problem (3 points): use a CAS device for the following. Turn in a pdf file as specified in the syllabus. **Matlab notes for this lab follow the brief answers.**

    Use Euler's method and then ode45 (fourth-order Runga Kutta) to estimate the solution to $x' = 1 + x^2 - t^3$ if $y(0) = -1$ and $0 \le t \le 2$. Print out the estimate for $x(2)$ using Euler's method with step size equal to $h = 0.5$, and then print out the estimate for $x(2)$ using ode45. ode45 will choose its own step sizes. Finally, make a graph of both estimated solutions for $0 \le t \le 2$ superimposed over the slope field of the differential equation.

Brief answers

1. Approximately 36.67° F ; 3.06 minutes.
2. Approximately 82.1 sec; approximately 145.7 sec.
3. $y = \dfrac{1}{2e^x + x + 1}$
4. $y = x\tan(\ln|x| + C)$
5. $y = (x + Cx^2)^{-3}$
6. $y = x(3\ln(x) + 1)^{1/3}$
7. $\frac{1}{3}x^3 + x^2y + xy^2 - y = \frac{4}{3}$
8. $x^3e^{x^2+y} - 4y^3 + 2x^2 = C$
9. $y = C_1e^{(-2+\sqrt{2})x} + C_2e^{(-2-\sqrt{2})x}$
10. $y = C_1e^{3x} + C_2xe^{3x}$
11. $y = e^{-x/4}\cos\left(\frac{\sqrt{7}}{4}x\right) - \sqrt{7}e^{-x/4}\sin\left(\frac{\sqrt{7}}{4}x\right)$
12. $y(x) = \dfrac{2(a-b)}{5}e^{-1.5x} + \dfrac{3a+2b}{5}e^x$
13. $z(t) = 2e^{-t}\cos(t)$
14. $y = C_1e^x + C_2x^3 + C_3x^2 + C_4x + C_5$
15. a) $r^3 - 3r^2 + 4r - 12 = 0$ b) $y''' - 3y'' + 4y' - 12y = 0$ c) $y = C_1e^{3x} + C_2\sin(2x) + C_3\cos(2x)$
16. $y'' = y$ Proof: either show the hyperbolics are independent and satisfy the equation, or show that the span of the exponentials equals the span of the hyperbolics.
17. Yes! Use the Wronskian to justify this answer.

MatLab Notes start on the next page.

Here is an example similar to your homework question.
The equation is x'=t-0.01x^2, x(-1)=0.2; I want to estimate x(1).

```
F=@(t,x) t-0.01*x.^2;

y0=.2;
[t,x]=ode45(F,[-1 1],y0);
disp('Using ode(45), the estimate for x(1) is '), x(45)
```

Using ode(45), the estimate for x(1) is ans = 0.1992

```
xe=ode1(F,-1,0.5,1,.2);

disp('Using eulers method, the estimate for x(1) is '), xe(5)
```

Using eulers method, the estimate for x(1) is

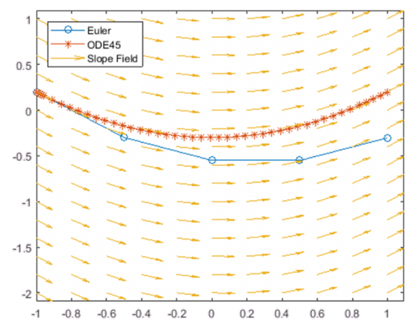
ans = -0.3037

```
plot(-1:0.5:1,xe,'o-')

hold on plot(t,x,'*-')

[T,X]=meshgrid(-1:0.2:1, -2:0.2:1); U =
ones(size(T));
V = F(T,X);
L=sqrt(1+V.^2); quiver(T,X,U./L,V./L,0.5)

axis tight
legend('Euler','ODE45', 'Slope
Field','Location','northwest')
```




```
function yout=ode1(F,t0,h,tfinal,y0)
% ODE1 A simple ODE solver.
% yout=ODE1(F,t0,h,tfinal,y0) uses Euler's
% method with fixed step size h on the interval % t0 <= t <= tfinal
% to solve
% dy/dt=F(t,y)
% with y(t0)=y0.
y=y0;
yout=y;
for t=t0:h:tfinal-h

s=F(t,y); y=y+h*s; yout=[yout;y];

end end
```